Source: *Proceedings of International Conference on Advances and New Challenges in Earthquake Engineering Research (ICANCEER02)*, Hong Kong, China, August 19-20, 2002.

AN ENERGY FRAMEWORK FOR DECENTRALIZED MARKET-BASED STRUCTURAL CONTROL

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ABSTRACT

The current state-of-practice in structural control uses the centralized Linear Quadratic Regulator (LQR) approach to determine optimal control forces. The centralized architecture of the LQR control approach does not easily scale to complex systems characterized by high sensor and actuator densities. In this paper, a decentralized control approach is proposed for application in structural control systems. Termed energy market-based control (EMBC), the control system's decision process is modeled after the laws of supply and demand that govern free-market economies. The demand function of system actuators reflect the real-time kinetic and strain energy response of the structural system while the supply function of the system power sources are influenced by the structure's external input energy. The approach is shown to yield control results comparable to those obtained from the centralized LQR solution.

INTRODUCTION

The concept of using control systems in the structural engineering domain has been proposed for about three decades (Yao 1972). In 1989, the first building to use an active-control system for limiting structural deflections during wind and seismic loadings was constructed (Kobori et al. 1991). Over twenty buildings, primarily in Asia, have since been constructed using a variety of structural control system designs (Nishitani 1998). Early control system types are termed active because one or two large actuators apply control forces to the structure directly. While success was attained using active control systems, they suffered from many technological limitations such as high costs and high power consumption demands. In response to these limitations, most recent research efforts have been centered upon the design of semi-active control systems. In semi-active control, passive energy devices are modified to possess variability in their response properties for service as indirect system actuators. Examples of semi-active control devices include stiffness control devices and variable dampers (Takahashi et al. 1998). Semi-active control devices are reliable, compact, require power on the order of tens of watts, and are cheaper to manufacture and operate (Symans and Constantinou 1999).

Innovation is driving control device designs towards smaller, cheaper, and more power efficient actuators for structural control. The technological improvement gained in adopting semi-active control serves as an example of this evolutionary trend. The trend suggests that control systems of the future will potentially employ up to hundreds of actuators and sensors, resulting in a large-scale control problem defined by high actuation and sensing densities. Current state-of-practice employs a centralized controller for the calculation of control forces based upon measurements obtained from system sensors. A centralized controller is generally not adopted in large-scale control systems because control force computations increase at faster than a linear rate with increases in system dimensionality (Lunze 1992).

As an alternative to the centralized controller, decentralized control techniques can be considered for adoption in large-scale control problems. Application of decentralized control solutions result in the reduction of the global system into an interrelated collection of smaller subsystems. A variety of decentralized control approaches can be considered for adoption in a structural control system. Modifications of the centralized linear quadratic regulator (LQR) in order to produce optimal decentralized controllers for structural control have been explored (Lynch and Law 2002). From a performance standpoint, it has been shown that no penalty is incurred in choosing a decentralized control solution.

Most recently, the explosion in development of MEMS sensing and actuation systems has resulted in many large-scale control problems. With the reliability of MEMS sensors and actuators lower than conventional counterparts, adaptive and flexible control methods are required with decentralized control solutions most popular. Researchers have explored using free-market concepts as one approach for controlling large-scale MEMS systems (Guenther et al. 1997). By modeling the control system as a free-market economy, where actuators are market buyers and power source are market sellers, an optimal control solution can result. Market-based control (MBC) methods have also been applied for controlling the computational load of microprocessors and for load-balancing in data networks (Clearwater 1996).

Lynch and Law (2001) have proposed a market-based control (MBC) approach specific to application in a structural control system. The MBC method was derived using linear demand and supply functions for behavioral definition of market participants. Although excellent control performance was attained, the linear market functions lacked a physical rationale. The scope of this research is to revisit the MBC derivation and to develop a more rational framework by incorporating the naturally occurring measures of energy in the system. Termed energy market-based control (EMBC), the approach will be tested using the semi-active controlled Kajima-Shizuoka building as an illustrative example. The EMBC control performance will be compared to that of the centralized LQR controller.

OVERVIEW OF MARKET-BASED CONTROL

The free market economies are an efficient means of allocating amongst market participants scarce resources, such as labor and goods. Free markets are decentralized in the *a priori* sense because the market mechanisms operate without knowledge of the global system. The historically poor performance of centralized economies underscores the efficiencies of the decentralized marketplaces. The competitive mechanisms of a free market can be extended for application to the control paradigm.

First, the marketplace is defined by a scarce commodity such as control power, control forces, or control energy, just to name a few potential quantities. In the control marketplace, the role of market buyers and sellers are assumed by system actuators and power sources respectively. The behavior of buyers is defined by individual utility functions, U_B, that measure the amount of utility derived by the buyer from purchasing the market commodity. Utility is a function of the price per unit commodity, p, the amount of commodity purchased, C_B, and response measures of the dynamic system, y. Similarly, the sellers are governed by individual profit functions, Π_S, that measure the amount of profit derived by the seller from selling the commodity. Profit is modeled as a function of the price per unit commodity, p, and commodity sold, C_S.

The goal of market buyers is to maximize their utility. In doing so, maximization of their utility functions is constrained by limiting the total purchase cost, pC_B, to be less than their instantaneous wealth, W. Maximization of the market sellers' profit functions is constrained by the maximum amount of commodity they possess, C_{MAX}.

$$\begin{aligned} &\max \Pi_{S1}(C_{S1}, p) \text{ subject to } C_{S1} \le C_{MAX1} \\ &\max \Pi_{S2}(C_{S2}, p) \text{ subject to } C_{S2} \le C_{MAX2} \\ &\qquad\vdots \\ &\max U_{B1}(C_{B1}, p, y_{B1}(t)) \text{ subject to } pC_{B1} \le W_1 \\ &\max U_{B2}(C_{B2}, p, y_{B2}(t)) \text{ subject to } pC_{B2} \le W_2 \end{aligned} \tag{1}$$

The simultaneous optimization of the utility and profit equations is viewed as a static optimization of the decentralized marketplace. Directly resulting from the static optimization are the demand functions of market buyers and the supply functions of market sellers.

The marketplace aggregates the demand functions of the individual buyers to obtain the global demand function of the market. In a likewise manner, the market aggregates the supply functions of all sellers to determine the market's global supply function. At each point in time, the demand function and supply function of the market share a point where they intercept. This equilibrium point represents the state of competitive equilibrium of the system thereby setting the equilibrium price of the commodity. With the equilibrium price found, the static optimization of the marketplace is complete and a transfer of commodity can exist between the market sellers and market buyers. This solution is termed Pareto optimal in the multi-objective optimization sense. Pareto optimal is defined by a market in competitive equilibrium where no market participant can reap the benefits of higher utility or profits without causing harm to other participants when a resource allocation change is made (Mas-Colell, Whinston, and Green 1995).

During an external excitation to the system, the marketplace goes into action with the market re-optimized at each time step for determination of an efficient control solution. The demand and supply functions change in time, necessitating a reevaluation of the marketplace at each time step with individual demand and supply functions aggregated and equated. When an equilibrium price is determined, all market buyers purchase their desired commodities and transfer wealth to the market sellers. The amount of commodity purchased by the market buyers (actuators) is converted into control forces to be applied to the structure. After money has been exchanged, the money obtained by the market sellers is evenly distributed back to the market buyers to represent income for their future purchases.

STRUCTURAL ENERGY DURING VIBRATIONS

The energy balance of a structural system during a seismic disturbance can easily be derived. First consider the equation of motion of an n degrees-of-freedom structural system subjected to a seismic disturbance and using controls to limit responses that would result:

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{D}\boldsymbol{u}(t) \tag{2}$$

The displacement response vector of the system is $\boldsymbol{x}(t)$, the control forces applied to the system by m actuators are represented by $\boldsymbol{u}(t)$, and the absolute displacement is $\boldsymbol{y}(t)$. The absolute displacement of the system, $\boldsymbol{y}(t)$, is simply the input ground displacement, $x_g(t)$, added to each term of the relative displacement vector, $\boldsymbol{x}(t)$. The mass, damping, and stiffness matrices are n x n in dimension and are denoted by $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$, respectively. It is assumed that the mass, damping and stiffness matrices are symmetric. $\boldsymbol{D}$ is the n x m location matrix for the application of control forces.

Equation (2) represents the equilibrium balance of forces in the structural system at any point in time. Integrating the forces over the response path from the initial position, $\boldsymbol{x}_o$, to the final position, $\boldsymbol{x}_f$, yields the energy of the balanced system (Wong and Yang 2001).

$$\int_{x_o}^{x_f} \ddot{\boldsymbol{y}}^T \boldsymbol{M} d\boldsymbol{x} + \int_{x_o}^{x_f} \dot{\boldsymbol{x}}^T \boldsymbol{C} d\boldsymbol{x} + \int_{x_o}^{x_f} \boldsymbol{x}^T \boldsymbol{K} d\boldsymbol{x} = \int_{x_o}^{x_f} \boldsymbol{u}^T \boldsymbol{D}^T d\boldsymbol{x} \tag{3}$$

The first term on the left-hand side of Equation (3) reflects the kinetic energy of the system while the third term represents the strain energy of the system. Both measures of energy are based upon conservative forces and are therefore path independent. Their measure is only dependent upon the current position and initial position of the system. Assuming the system is initially at rest, the kinetic and strain energy of the system can be rewritten and Equation (3) updated.

$$\frac{1}{2}\dot{\boldsymbol{y}}^T \boldsymbol{M}\dot{\boldsymbol{y}} + \int_{x_o}^{x_f} \dot{\boldsymbol{x}}^T \boldsymbol{C} d\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^T \boldsymbol{K}\boldsymbol{x} - \int_{x_o}^{x_f} \boldsymbol{u}^T \boldsymbol{D}^T d\boldsymbol{x} = \int_{x_o}^{x_f} \ddot{\boldsymbol{y}}^T \boldsymbol{M} d\boldsymbol{x}_g \tag{4}$$

The four terms of the left-hand side of Equation (4) represent respectively, kinetic energy (KE), damping energy (DE), strain energy (SE) and control energy (CE). These four energies balance the input energy (IE) resulting from the ground motion as shown on the right-hand side of Equation (4).

DERIVATION OF ENERGY MARKET-BASED CONTROL

The derivation of energy market-based control (EMBC) is centered upon a marketplace allocating the scarce commodity of control energy. The method begins with the selection of demand and supply functions that reflect measures of energy in the system. The demand and supply functions will each contain a "tuning" constant that can be used to vary their sensitivities.

Derivation of Demand and Supply Functions

In the energy marketplace, the scarce commodity of control energy is used to determine the magnitude of control forces applied to the structural system. The form of the demand function is selected to reflect two intentions of the market buyers. First, when the price of control energy is zero, the demand of the market buyer is equal to the input energy of its degree-of-freedom. Second, the demands of the

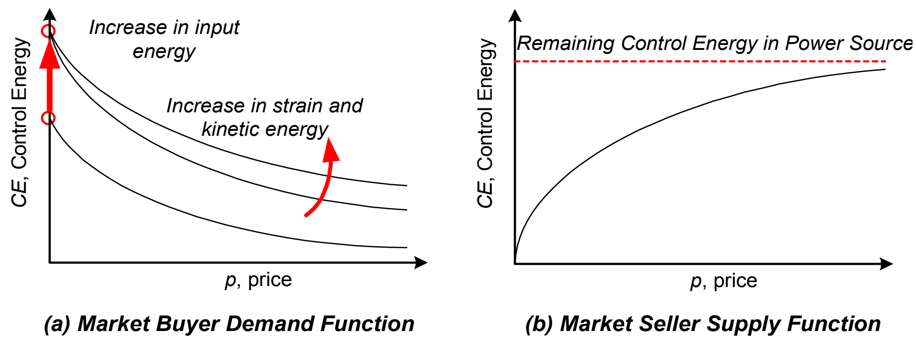


Figure 1 - Energy market-based control (EMBC) demand and supply functions

market buyers asymptotically converge toward zero at infinite prices. To encapsulate these two characteristics, an exponential demand function for the i^{th} market buyer, is proposed:

$$CE_i = W_i \left| \ddot{y}_i(t) m_i dx_g \right| e^{\frac{-2p\alpha}{\dot{y}_i^2 m_i + x_i^2 k_i}} \tag{5}$$

The y-axis intercept is equal to the instantaneous input energy of the ground motion at a particular degree-of-freedom multiplied by the market buyer's wealth, W_i. The exponential decay of the demand function is dependent upon the kinetic and strain energy of the system as depicted in the denominator of the exponential term. As the response of the system increases due to greater kinetic and strain energy, the rate of decay decreases. The tuning constant, α, is provided to control the sensitivity of the demand function. Figure 1(a) illustrates the behavior of the modeled demand function.

The control system's battery sources represent the market sellers whose actions are described by supply functions. Each market seller has in its possession a certain amount of control energy. Again, two observations of the market seller's behavior are required before specifying a suitable supply function. First, if the price of power is set to zero, no market seller is willing to sell. Second, as the price grows to infinity, each market buyer would be willing to sell all of its remaining control energy denoted by L_i. As a result, the following supply function is proposed:

$$CE_i = L_i \left(1 - e^{-\beta p}\right) \tag{6}$$

Equation (6) provides an origin intercept in addition to an asymptotic convergence to the remaining battery life at very large market prices. The constant β is used to provide a means of adjusting the supply function. Figure 1(b) presents a graphical interpretation of the market seller supply function.

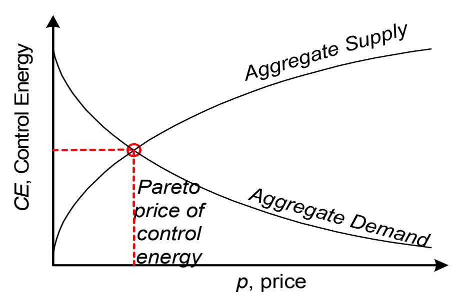


Figure 2 - Determination of the competitive equilibrium price of control energy

Equilibrium Price of Power

With the demand and supply functions for all market participants established, the Pareto optimal price at each time step can be readily determined. The aggregate demand function is set equal to the aggregate supply function to determine the competitive equilibrium price of energy for a given time step. A graphical interpretation, as shown in Figure 2, is the price of control energy of the point where the global demand and supply functions intersect. It can be shown that this intersection point always exists. The solution represents a Pareto optimal price of control energy for the marketplace.

The amount of control energy that is purchased by each system actuator is used to determine the applied control force. Given the instantaneous control energy purchased by an actuator, the control force u_i can be determined from Equation (7).

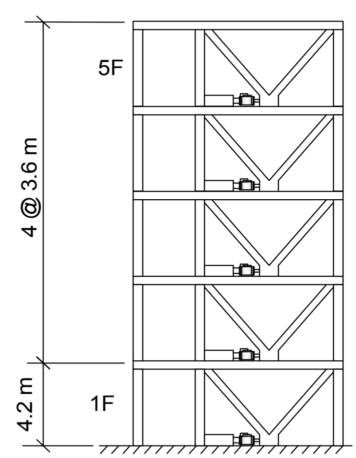
$$CE = u_i \Delta x_i \qquad (7)$$

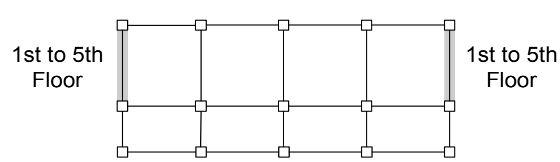
After control energy has been purchased, the energy is evenly subtracted from the system power sources. Similarly, the amount of energy purchased by an actuator times the market price per unit power determines the amount of wealth removed from each actuator's total wealth.

EXAMPLE EMBC IMPLEMENTATION IN THE KAJIMA-SHIZUOKA BUILDING

The Kajima-Shizuoka Building is used to illustrate the implementation of the derived EMBC control solution. The structural details of the building are presented in Figure 3 (Kurata et al. 1999). A total of ten semi-active hydraulic dampers, capable of changing their damping coefficient in real-time, are installed in the structure's weak longitudinal direction. Each SHD control device is capable of producing a maximum control force of 1,000 kN.

To quantify the performance of the EMBC solution, the structure is controlled for the El Centro, Taft, and Northridge seismic disturbances. For the three earthquake records selected, peak absolute ground velocities have been normalized to a value of 50 cm/sec. The performance of the EMBC controller will be directly compared to that of a centralized LQR controller.





Floor	Seismic Mass (kg)	Story Stiffness (kN/m)
1	215,200	147,000
2	209,200	113,000
3	207,000	99,000
4	204,800	89,000
5	266,100	84,000

Figure 3 – The Kajima-Shizuoka Building, Shizuoka, Japan

For the implementation of the EMBC controller, the demand and supply constants, α and β, are set to unity. It is determined that values of unity for these two constants makes the supply and demand functions sufficiently sensitive to yield excellent control results. Each floor of the structure that contains two actuators is provided with an equal amount of initial wealth.

$$W_1 = 1000; \quad W_2 = 1000; \quad W_3 = 1000; \quad W_4 = 1000; \quad W_5 = 1000 \tag{8}$$

The total amount of power initially provided by the system power source is roughly calculated based upon the observation that the control system battery in the Kajima-Shizuoka Building is designed to last for 8 continuous minutes with 10 SHD devices each drawing 70 W of power. As a result, the total energy provided to the system battery sources is set to 1.25 x 10^{10} J.

$$L_T = 1.25 \quad x \quad 10^{10} \quad J \tag{9}$$

An LQR controller is also implemented for the structure. The ***Q*** and ***R*** weighting matrices of the LQR controller are chosen to weigh with heavier emphasis on the absolute velocity response of the system degrees-of-freedom.

$$\boldsymbol{Q} = [0 \quad \boldsymbol{I}] \text{ and } \boldsymbol{R} = 1x10^{-13}[\boldsymbol{I}] \tag{10}$$

Figure 4 presents the maximum absolute interstory drift of the Shizuoka Building when no control is used and when the LQR and EMBC controllers are employed. As shown, both the LQR and EMBC controllers are effective in reducing the drift response of the structure, with minimal differences between the two control performances. Only for the Northridge seismic disturbance does the LQR controller exhibit slightly superior performance when compared to the drift response of the EMBC controller at the 2nd and 3rd stories.

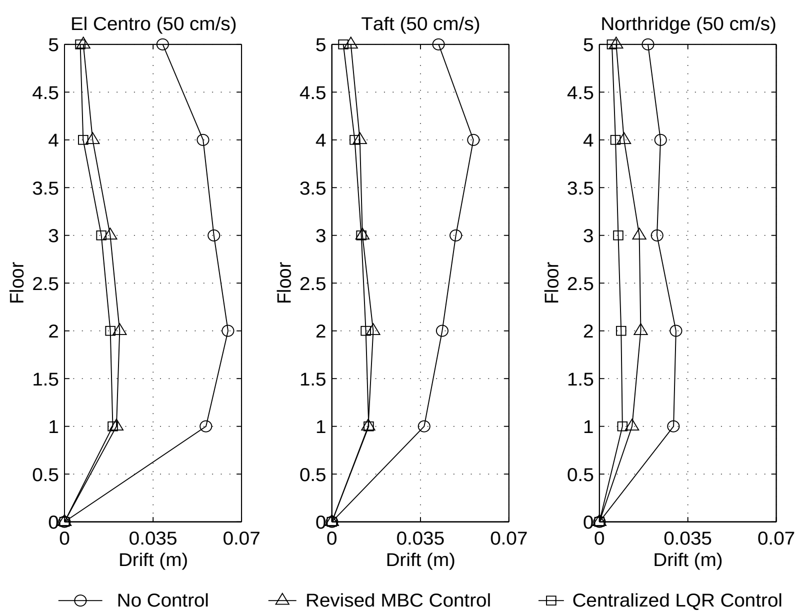


Figure 4 - Maximum absolute interstory drift using LQR and EMBC controllers

CONCLUSION

The scope of this research focused upon extending the concepts of decentralized market-based control (MBC) using an energy framework. The decentralized EMBC controller was implemented in the Kajima-Shizuoka Building with excellent control results observed compared to the centralized LQR controller. Other market models may exist and provide more superior control performances.

ACKNOWLEDGEMENTS

This research is partially sponsored by the National Science Foundation under Grant Numbers CMS-9988909 and CMS-0121842.

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