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A Market-Based Control Solution for Semi-Active Structural Control

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Abstract

A decentralized control algorithm for structural control system design has been developed. This novel approach is termed market-based control. As a potential alternative to the classical Linear Quadratic Regulator (LQR), this method decentralizes the decision process of the control system and allows for independent and autonomous control formulation to occur directly upon the control device. This paper outlines the theoretical formulation of both approaches and presents their performance on a semi-active controlled structural model.

Introduction

Early research efforts in the field of structural control concentrated heavily upon active structural control systems. These systems limit structural deflections by employing actuators to apply forces directly to the structure. While remarkable progress was made, it was determined that the approach had many technological and economic limitations when used in protecting structures during earthquakes. To overcome these limitations, a semi-active approach to structural control was formulated. In this elegant approach, actuators are no longer used to apply forces to a structure directly. Rather, the forces needed for control are generated indirectly by devices that change the overall damping and stiffness properties of the structure. With small energy consumption characteristics, semi-active devices are an especially attractive solution for limiting earthquake deflections.

Numerous semi-active devices have been proposed and constructed. In particular, researchers at Kajima Corporation, Japan, have successfully designed and tested a variable damping device known as the Semi-active Hydraulic Damper

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(SHD) [Kurata *et al*., 1999]. The SHD device is a variable damper whose damping coefficient can be changed by changing the orifice opening between the two hydraulic chambers of the damper. Joint research efforts between the University of Notre Dame and Washington University have produced a variable damping device known as the magnetorheological damper [Spencer *et al*., 1998]. The damping coefficient of the magnetorheological damper changes when a magnetic field around the piston chamber causes a viscosity change of the damper's fluid. The SHD device can provide a maximum damping force of 1000 kN using only 70W of power while the magnetorheological damper can provide a 200 kN damping force using 20 to 50W of power.

In practice, a variable damping device is strategically placed at the apex of a lateral resisting frame's V-brace. With dampers located at various floors throughout the structure, they are centrally controlled by a controller located within the structure. Kajima has recently installed SHD devices in a five story structure in Shizuoka, Japan with an SHD device at the first four floors [Kurata *et al*., 1999]. Accelerometers located upon each floor feedback information of the structure's dynamics to the controller which uses a traditional Linear Quadratic Regulation (LQR) algorithm to generate commands for the control devices.

It is certain that semi-active devices will continue to evolve into more compact, cheaper and efficient control devices. With the arrival of small and inexpensive control devices, structures will be able to deploy hundreds of these devices for vibration control during earthquakes. The means of centrally controlling each semi-active device will no longer be an economically efficient solution for systems with an abundance of devices. As an alternative, a decentralized control method needs to be formulated for the control of structures that employ a large number of control devices. In such an approach, a central controller will no longer be needed to regulate the structure during an excitation since each semi-active device will have on-board computational means for formulating a semi-optimized control solution. By decentralizing the control solution, the control algorithm will become model independent. This will lead to robust control of the structure if device or structural failure was to occur.

The Classical LQR Technique of Structural Control

Structures are often idealized as one-dimensional lumped mass shear models in which each floor of the structure represents one degree of freedom. Under an excitation, the structure's dynamics are defined by the response of the system at each degree of freedom. In control vernacular, such a system is often referred to as a multi-input multi-output system (MIMO). Transforming the dynamics of the system from the continuous time domain to the complex plane via Laplace transforms, the dynamic response of the system can be represented by the roots of the characteristic equation of the system. These roots, often termed the poles of the system, will generally fall in the left half side of the complex plane and correspond to the various modes of response of the structure. Figure 1 shows a general one degree of freedom structure represented in the complex plane.

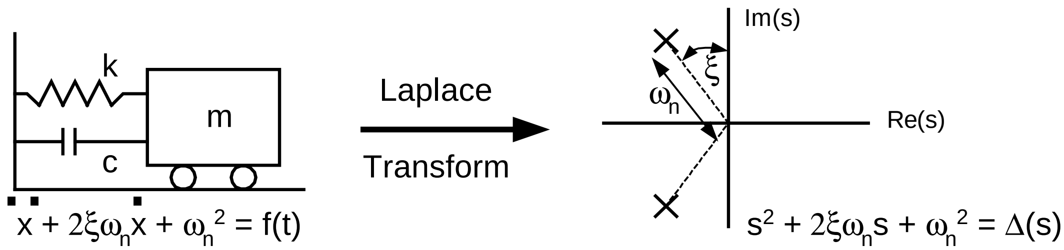

Figure 1 – Physical System Converted to the Complex Plane

To improve the response of the structure to dynamic disturbances, a controller is employed which causes the movement of the system's roots to more desirable locations. In MIMO systems, pushing roots to a desired location can be an arduous procedure when considering the control effort necessary to drive the roots to those locations. To handle this complexity, optimal control techniques have been developed, such as LQR, which are a systematic guide of pole placement that allows for weighing the control response against control effort [Franklin *et al*., 1990].

Optimal control techniques are a powerful tool for determining optimal control performance through the minimization of a cost function of the system. The form of the cost function is flexible and can contain terms that represent different weighted costs of system parameters. The cost function for LQR optimal control is

$$J = \int_0^t \left\{ X^T(\tau) Q X(\tau) + U^T(\tau) R U(\tau) \right\} d\tau$$

The state vector used in the cost function is from a one dimensional ordinary differential equation representation of the system. The state vector *X(t)* contains both the displacement vector of the structure, *x(t)*, and the velocity vector *v(t)*. The control vector of the system is represented by *U(t)*. The matrices *Q* and *R* represent weighting upon the energy associated with the structural response and the control input respectively.

To ensure that a minimum of the cost function, *J*, can be found, the weighting matrices *Q* and *R* must be positive definite so that the n-dimensional shape of the cost function's surface is upward convex and a global minimum point of the surface exists. Proof of a minimum point can be given by considering the Taylor-series expansion of the cost function about the minimum point [Stengel, 1994]. The response and control terms of the cost function are quadratic to ensure that the cost function's surface has a defined slope at all points. With no cusp points, a minimum in the overall cost function is guaranteed given the positive definite condition on *Q* and *R*. In a one dimensional example, consider if the cost is proportional to the system displacement. The cost function of the displacement will have no slope defined at the zero point since a cusp is present. Furthermore, a quadratic cost function has the additional benefit of placing harsher penalties on large displacements than small displacements. A perfect analogy would be a missile is still considered successful if it misses the center of the target by a small amount but is unsuccessful if it misses the target by a great amount. Figure 2 continues to illustrate the rational of the LQR method in one dimension.

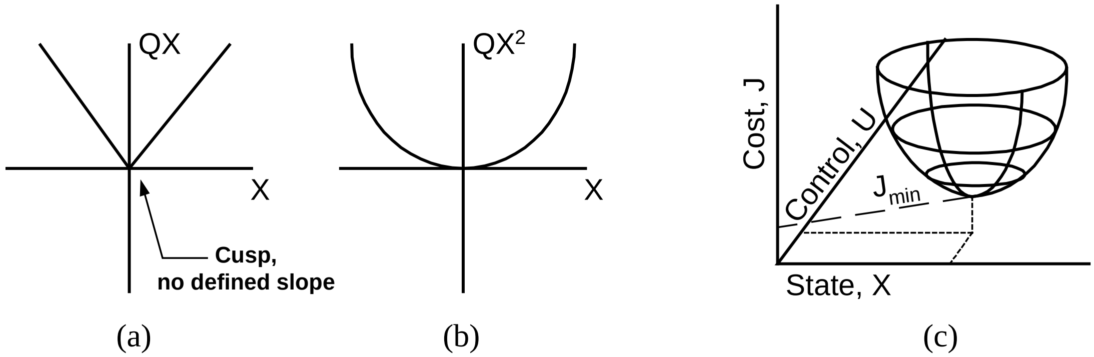

Figure 2 – An Illustration of the LQR Optimal Control Technique, (a) Linear Cost on the State has no Definable Minimum, (b) Quadratic Cost on the State has a Definable Minimum, (c) Cost Function Surface and Minimization

The minimum of the cost function is found by adjoining the cost function, *J*, and the constraining equation of motion of the system with a time dependent Lagrangian multiplier, $\lambda(t)$. Neglecting the loading imposed upon the system and assuming the Lagrangian multiplier is proportional to the state vector by the Riccati matrix, *P(t)*,

$$\lambda(t) = P(t)X(t)$$

an algebraic solution exists for the Riccati equation that gives the optimal control solution, *u(t)* in reference to the weighting between matrices *Q* and *R*. The solution is found to be proportional to the state of the system by matrix *G*, with new pole locations encapsulated within the controller, *G*.

$$u(t) = -\frac{1}{2}R^{-1}B^{T}PX(t) = GX(t)$$

The entire formulation of the LQR optimal control technique is dependent upon complete knowledge of the system properties. Furthermore, the system requires feedback of the state of the system in the form of the displacement and velocity at each degree of freedom. However, in common practice, accelerometers are the primary sensor in determining structural response. Unfortunately, this adds a level of complexity to the LQR method requiring special techniques to handle acceleration feedback control [Yedavalli *et al*., 1997].

Decentralized Market-Based Control

Given the environment of hundreds of semi-active control devices in use within a structure, a centralized control method is not practical and it is more feasible to divide up the responsibilities of the central controller and distribute them to each control device. Out of this environment of decentralization, interacting control devices and the introduction of a scarce resource does market-based control emerge.

A market is a collection of interacting agents with local intentions but who achieve an overall efficient global behavior. Consider free market economies where there exist buyers who seek at a local level to achieve the best buy for a product and producers whose local goal is to maximize profits. Such a system, free of centralized control, leads to an efficient allocation of resources in the economy. Due to the countless number of buyers and sellers making transactions, centralized economies are very difficult to control as proven by history. Not unique to real economies, a market style approach has been taken to control systems employing system resources such as computer memory or control resources in micro-electrical machine (MEMs) systems [Clearwater, 1996].

In this study, a market-based control solution is proposed for the control of a dynamically excited structure during an earthquake. Similar market-based proposals have been made for increasing the stability of columns from buckling under axial loads [Guenther *et al*., 1997]. Consider a market place of semi-active control devices where each control device makes decisions about the amount of control force applied based upon the laws of economics. Provided to each device is an amount of fictitious wealth that can be used by the device to purchase control power. The power used for control and the wealth each device possesses to purchase power are the two scarce resources of the economy.

Each device has demand for control power, P, depending upon the amount of utility, U, that can be attained from purchasing this power. Each device's utility is a function of the displacement, x_i, velocity, v_i, and the cost of a unit of power, p. Each device has wealth, w_i. One example utility function is:

$$\frac{U_i}{P_i} = \alpha|x_i| + \beta|v_i| - \frac{pP_i}{w_i}$$

In an attempt to attain the demand of each device, the associated utility function is maximized in relation to the amount of power used. As a result, a function of the amount of power demanded versus the price of a unit of power is obtained. The weighting coefficients α and β are used to allow the pricing model to be tuned to the structure.

$$P_i = \frac{2w_i\left(\alpha|x_i| + \beta|v_i|\right)}{p}$$

On the supply side of the system, the producer (*e.g.* the device's battery) seeks to maximize profit associated with selling power to the semi-active device. The profit, ρ, obtained by the producer is proportional to the power sold to the device, P_i, and is a function of the cost of power production, C, which is assumed to be proportional to the square of the power produced.

$$\rho = P_i\left(p - C\right)$$

To find the relationship between the amount of power, P_i, and the price of power, p, for the supplier, the profit is maximized in relation to the power used. Again, a weighting coefficient, η, is used to allow for the tuning of the supply model of the device. Such a coefficient can represent conservation of power versus control effect desired.

$$P_i = 2\eta p$$

Demand and supply is aggregated for all of the devices in the market place and an equilibrium price for power is found by finding the intersection point of the supply and the demand functions. The equilibrium price of power found is used to determine the amount of control power used by each device. Figure 3 gives a graphical interpretation of the market-base equilibrium price model. The equivalent LQR method can be thought of finding the minimum of the sum of the supply and demand functions while market-based control seeks the equilibrium point of the two.

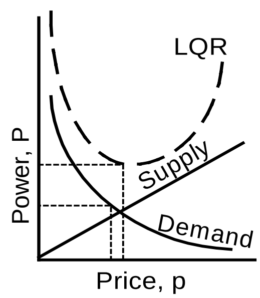

Figure 3 – An Illustration of the Market-Based Control Technique

At each step in time, a new market model is generated and a new equilibrium price of power is determined.   Wealth is conserved in the system and is distributed back to each control device equally after every time step.

Example – A Market-Based Control Solution for a Five-Story Structure

A five-story structure was controlled using both LQR optimal control and market-based control techniques. Figure 4 illustrates the one-dimensional structure considered for a comparison analysis of the two control techniques.

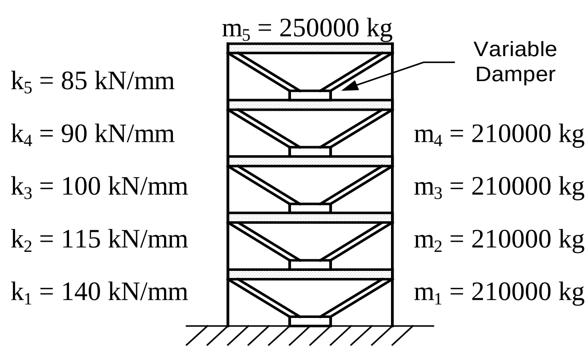

Figure 4 – Five-Story Structure with Semi-Active Variable Damping Devices

The variable dampers selected for this analysis are assumed to have properties identical to the SHD device designed and tested by Kajima. A maximum damping force of 1,000 kN can be generated by the damper. The minimum and maximum damping coefficients of the device are 1 kN sec/mm and 200 kN sec/mm respectively. In determining the control force desired, a check is performed to ensure that the control force is in a direction identical to the floor velocity. If it is not, no control force is applied and the orifice valve of the damper is fully open. If a desired control force exceeds either the maximum force of the device or the maximum damping coefficient times the current velocity, the control force is adjusted appropriately. In this model, it is assumed that there are two damping devices upon each floor of the structure. In the structural model considered, the structure is excited under the full scale El Centro N-S (1940) earthquake record.

The LQR control technique requires an iterative analysis to determine the optimal weighting associated with the Q and R matrices. Once the ideal weighting is attained, the controller matrix, G, is found. In a similar fashion, the weighting coefficients in the pricing models of the market-based system are adjusted to provide the best performance of the market. At the start of the analysis, each device is given an equal amount of market wealth. The maximum story displacements and the story drifts were considered as suitable parameters for comparing the two different control techniques. An uncontrolled structural model free of the SHD devices was also analyzed. Figure 5 compares the results of the three systems considered.

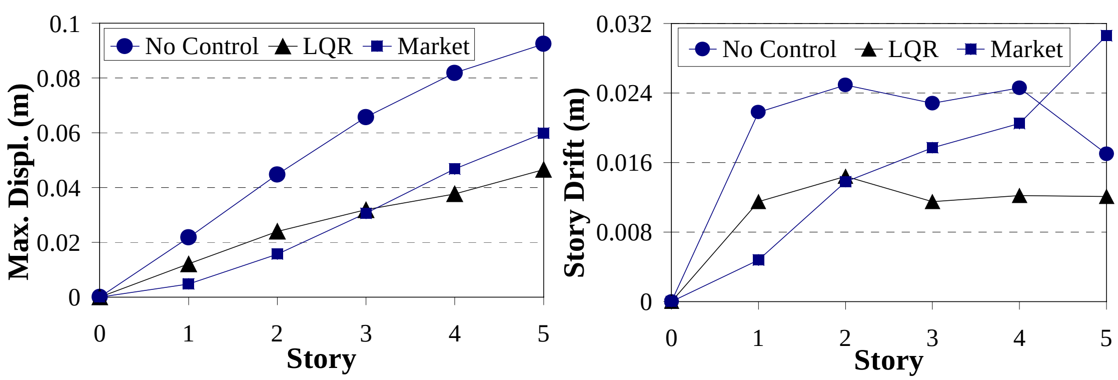

Figure 5 (a) Maximum Story Displacement, (b) Inter-story Drift

Conclusion

It is evident that the LQR method provides the best overall control of the system. However, market-based control is not far behind in terms of performance. Market-based control was more successful at the lower floors of the structure with maximum floor displacement and inter-story drift lower than those obtained from the LQR technique, while at the upper two stories, the LQR solution was better. Since there is no rigid formulation of the market-based control method, different demand and supply functions can be considered on a more problem specific level. This could lead to better functions that will yield the best benefits of the method. The goal of the market-based method is not to provide the optimal control solution but only a suitable solution in applications where optimal control techniques are not well suited.

The LQR method can be proven to render a dynamic system more stable through the implementation of the controller, *G*. Market-based methods need to be considered further in order to investigate issues associated with system stability in a closed form. Market-based control techniques are particularly well suited for a decentralized and distributed system. With a highly intuitive way of formulating the control problem, the method will allow engineers to propose control systems that can be easily expanded and adaptive.

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