Parallel Computing for Seismic Geotechnical Applications

Jinchi Lu[1], Ahmed Elgamal[2*], Kincho H. Law[3], and Zhaohui Yang[1]

[1]University of California, San Diego, Department of Structural Engineering, La Jolla, CA 92093-0085
[2*]Corresponding author, University of California, San Diego, Department of Structural Engineering, SERF Building Rm 345, Mail Code 0085, La Jolla, CA 92093-0085; PH (858)822-1075; FAX (858)534-1310; email: elgamal@ucsd.edu
[3]Stanford University, Department of Civil and Environmental Engineering, Stanford, CA 94305-4020

Abstract
Parallel computing is gradually becoming a main stream tool in geotechnical simulations. The need for high fidelity and for modeling of fairly large 3-dimensional (3D) spatial configurations is motivating this direction of research. A new program ParCYCLIC for seismic geotechnical applications has been developed. Salient characteristics of the employed parallel sparse solver will be presented. Using this code, simulations of seismically-induced liquefaction, lateral-spreading, and countermeasures will be presented and discussed.

Introduction

Large-scale finite element (FE) simulations of earthquake-induced liquefaction effects often require a lengthy execution time. Utilization of parallel computers, which combine the resources of multiple processing and memory units, can potentially reduce the execution time significantly and allow simulations of large and complex models that may not fit into a single processing unit.

Parallel computing is gradually becoming a main stream tool in geotechnical simulations. Bielak et al (2000) modeled earthquake ground motion in large sedimentary basins using a 3D parallel linear finite element program with an explicit integration procedure. They noted that the implementation of an implicit time integration approach is challenging on distributed memory computers, requiring significant global information exchange (Bielak et al. 2000). Yang (2002) developed a parallel finite element algorithm, i.e. Plastic Domain Decomposition (PDD), and attempted to achieve dynamic load balancing by using an adaptive partitioning-repartitioning scheme.

The research reported herein focuses on the development of a state-of-the-art nonlinear parallel finite element code (implicit time integration method employed) for earthquake ground response and liquefaction simulation. The parallel code, ParCYCLIC, is implemented based on a serial program CYCLIC, which is a nonlinear finite element program developed to analyze liquefaction-induced seismic response (Parra 1996; Yang and Elgamal 2002).

Finite Element Formulation

In CYCLIC and ParCYCLIC, the saturated soil system is modeled as a two-phase material. A simplified numerical formulation of this theory (Chan 1988), known as *u-p* formulation (in which displacement of the soil skeleton *u*, and pore pressure *p*, are the primary unknowns), was implemented in a 3D Finite Element program CYCLIC (Parra 1996; Yang 2000; Yang and Elgamal 2002).

The *u-p* formulation is defined by (Chan 1988): 1) the equation of motion for the solid-fluid mixture, and 2) the equation of mass conservation for the fluid phase that incorporates equation of motion for the fluid phase and Darcy's law. These two governing equations are expressed in the following finite element matrix form (Chan 1988):

$$\mathbf{M}\ddot{\mathbf{U}} + \int_{\Omega} \mathbf{B}^{\mathbf{T}} \boldsymbol{\sigma}' \, d\Omega + \mathbf{Q}\mathbf{p} - \mathbf{f}^{s} = \mathbf{0} \tag{1a}$$

$$\mathbf{Q}^{\mathbf{T}}\dot{\mathbf{U}} + \mathbf{S}\dot{\mathbf{p}} + \mathbf{H}\mathbf{p} - \mathbf{f}^{p} = \mathbf{0} \tag{1b}$$

where **M** is the total mass matrix, **U** the displacement vector, **B** the strain-displacement matrix, $\boldsymbol{\sigma}'$ the effective stress tensor, **Q** the discrete gradient operator coupling the solid and fluid phases, **p** the pore pressure vector, **S** the compressibility matrix, and **H** the permeability matrix. The vectors $\mathbf{f}^{s}$ and $\mathbf{f}^{p}$ represent the effects of body forces and prescribed boundary conditions for the solid-fluid mixture and the fluid phase, respectively. Equations 1a and 1b are integrated in the time domain using a single-step predictor multi-corrector scheme of the Newmark type (Chan 1988; Parra 1996).

Soil Constitutive Model

In ParCYCLIC, a plasticity-based formulation is employed. The constitutive model was developed with emphasis on simulating the liquefaction-induced shear strain accumulation mechanism in clean medium-dense sands (Elgamal et al. 2002; Yang and Elgamal 2002; Elgamal et al. 2003; Yang et al. 2003). Special attention was given to the deviatoric-volumetric strain coupling (dilatancy) under cyclic loading (e.g., Figures 1 and 2), which causes increased shear stiffness and strength at large cyclic shear strain excursions (i.e., cyclic mobility). The main elements allowing for cyclic mobility response include:

The yield surface (Figure 1) is defined by the Lade and Duncan (1975) function:

$$f = \frac{I_1^3}{I_3} - \kappa_1 = 0 \qquad (2)$$

where I_1 and I_3 are the first and third stress invariants respectively, and κ_1 (>27) is a parameter related to soil shear strength (or friction angle ϕ). In the context of multi-surface plasticity, a number of similar surfaces with a common apex form the hardening zone (Figure 1). The flow rule is chosen assuming associativity in the deviatoric plane. And a non-associative flow dictates shear-induced contraction and dilation.

The phase transformation (PT) surface (Ishihara et al. 1975) defines the boundary between contractive and dilative behavior (Figure 2) under shear loading. Along the PT surface, the stress ratio η ($= \tau / p'$, where τ is the octahedral shear stress and p' the effective mean confinement) is denoted as η_{PT}. Depending on the value of η with respect to η_{PT} and the sign of $\dot{\eta}$ (time rate of η), distinct contractive/dilative (dilatancy) responses are reproduced (Yang et al. 2003; Yang and Elgamal 2004).

Thus, under undrained conditions, the adopted flow rule defines the following phases of soil response (Figure 2):

1) The contractive phase inside the PT surface ($\eta < \eta_{PT}$, phases 0-1 and 4-5), as well as outside during shear unloading ($\eta > \eta_{PT}$ with $\dot{\eta}$<0, phase 3-4).
2) The dilative phase during shear loading, with the stress state outside the PT surface ($\eta > \eta_{PT}$ with $\dot{\eta} > 0$, phase 2-3), and
3) The neutral phase (phase 1-2 and 5-6) between the contraction (phase 0-1) and the dilation (phase 2-3) phases.

Parallel Implementation

Parallel Program Strategies

Programming architectures required to take advantage of parallel computers are significantly different from the traditional paradigm for a serial program (Law and Mackay 1993). ParCYCLIC employs the single-program-multiple-data (SPMD) paradigm, a common approach in developing application software for distributed memory parallel computers (Lu et al. 2004; Peng et al. 2004). In this approach, problems are decomposed using well-known domain decomposition techniques. Each processor of the parallel machine solves a partitioned domain, and data communications among sub-domains are performed through message passing. The SPMD model has been applied successfully in the development of many parallel finite element programs from legacy serial code (De Santiago and Law 1996).

Computational Procedures

The computational procedure of ParCYCLIC can be basically divided into two distinct phases: the initialization phase and the nonlinear solution phase. The initialization phase consists of reading input files, performing mesh partitioning and symbolic factorization. METIS (Karypis and Kumar 1997), which is a set of libraries for graph partitioning developed at the University of Minnesota, is used to partition the finite element mesh at this phase. An automatic domain decomposer for performing domain decomposition, based on the METIS ordering, is also implemented in ParCYCLIC.

In the nonlinear solution phase, the modified Newton-Raphson algorithm is employed. When needed, a time-step splitting algorithm is employed to achieve convergence.

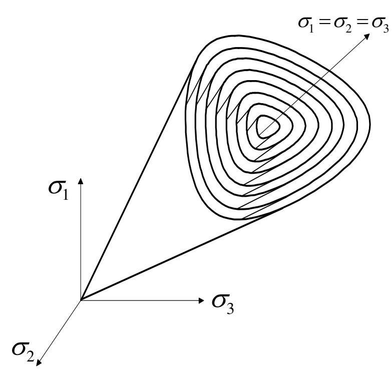

Figure 1. Configuration of multi Lade-Duncan yield surfaces in principal stress space (Yang and Elgamal 2004).

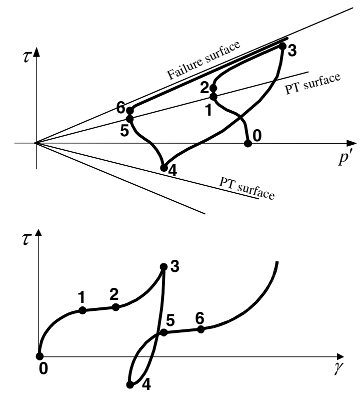

Figure 2. Schematic showing the model undrained effective stress path and shear stress-strain response (Yang et al. 2003).

Parallel Sparse Solver

Nonlinear finite element computations of earthquake simulations involve the iterative solution of sparse symmetric systems of linear equations (Lu et al. 2004; Peng et al. 2004). Solving the linear system is often the most computationally intensive task, especially when an implicit time integration scheme is employed. ParCYCLIC employs a direct sparse solution method proposed and developed by Law and Mackay (1993). The concept of the sparse solver incorporated in ParCYCLIC is briefly described below.

Given a linear system of equations $Kx = f$, the symmetric sparse matrix K is often factored into the matrix product LDL^T, where L is a lower triangular matrix and D is a diagonal matrix. The solution vector x is then computed by a forward solution, $Lz = f$ or $z = L^{-1}f$, followed by a backward substitution $DL^Tx = z$ or $x = L^{-T}D^{-1}z$. Sparse matrix factorization can be divided into two phases: symbolic factorization and numeric factorization (Law and Mackay 1993). Symbolic factorization determines the structure of matrix factor L from that of K (i.e. locations of nonzero entries). Numeric factorization then makes use of the data structure determined to compute the numeric values of L and D. The nonzero entries in L can be determined by the original nonzero entries of K and a list vector, which is defined as:

$$PARENT(j) = \min\{i \mid L_{ij} \neq 0\} \qquad (3)$$

in which j is the column number and i the row subscript. The array *PARENT* represents the row subscript of the first nonzero entry in each column of the lower matrix factor L. The definition of the array *PARENT* results in a monotonically ordered elimination tree T of which each node has its numbering higher than its (Law and Mackay 1993). Furthermore, by topologically postordering the elimination tree, the nodes in any subtree can be numbered consecutively. The resulting sparse matrix factor is partitioned into block submatrices where the columns/rows of each block correspond to the node set of a branch in T. Figure 3 shows a simple finite element grid and its post-ordered elimination tree representation.

For parallel implementation of the sparse matrix factorization, the processor assignment strategy can be based on matrix partitioning according to the post-ordered elimination tree. Essentially, the strategy is to assign the rows corresponding to the nodes along each branch (column block) of the elimination tree to a processor or a group of processors (Figure 3).

The parallel numerical factorization procedure is divided into two phases (Law and Mackay 1993). In the first phase, each processor independently factorizes certain portions of the matrix assigned to a single processor. In the second phase, other portions of the matrix shared by more than one processor are factored. Following the parallel factorization, the parallel forward and backward solution phases proceed to compute the solution to the global system of equations.

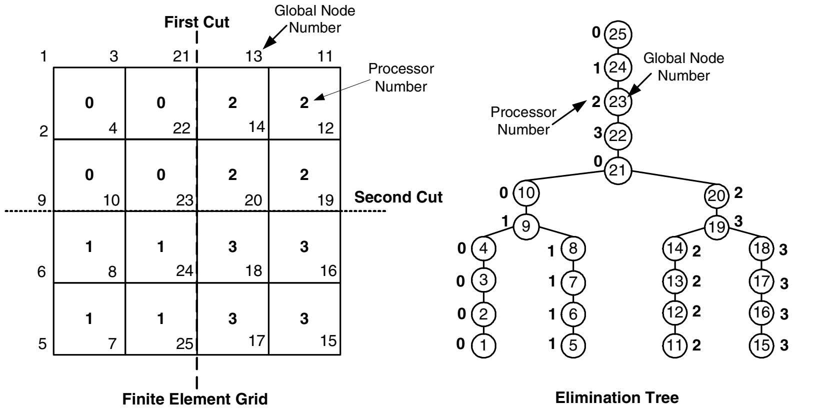

Figure 3: A finite element mesh and its elimination tree representation (Law and Mackay 1993).

Parallel Performance
ParCYCLIC has been successfully ported on many different types of parallel computers and workstation clusters, including IBM SP machines and Linux workstation clusters. The parallel performance was evaluated by simulating a stone column centrifuge test model on the Blue Horizon machine (IBM SP) at San Diego Supercomputer Center (SDSC). In this stone column model (half mesh shown in Figure 4), a number of gravel columns are embedded into a fully-saturated silt soil stratum. Figure 5 displays the speedup and the execution times for the nonlinear solution phase for one time step. Excellent parallel speedup is achieved, as shown in Figure 5. Note that the stone column model, with a scale of 364,800 degrees of freedom (dofs), cannot fit into the memory of less than 4 processors.

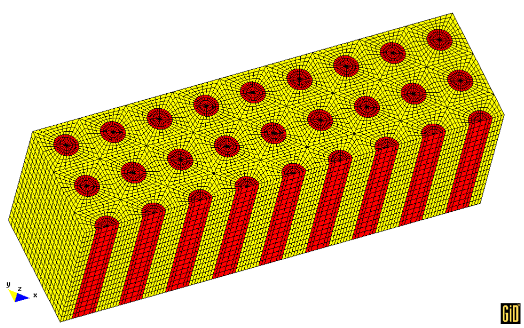

Figure 4. Finite element model of a stone column centrifuge test.

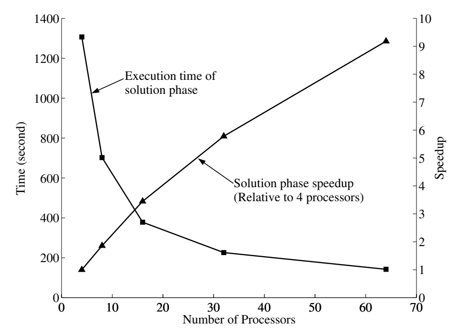

Figure 5. Execution times and speedup of the nonlinear solution phase (one time step) for the stone column model.

Numerical Simulation of Shallow Foundation Settlement and Remediation

A 10 m saturated medium sand layer is studied (calibrated based on Nevada sand at about D_r = 40%). In view of symmetry, a half-mesh (5,320 elements and 80,118 degrees of freedom in total) was used as shown in Figure 6. Herein, the load (40 kPa, about 2 m of an equivalent soil overburden) is simply applied at ground level in the form of a distributed surficial vertical stress over a 10 m x 10 m area (the dark zone at the ground surface in Figure 6 also shows this foundation size). At any given depth, displacement degrees of freedom of the left and right boundaries were tied together (both horizontally and vertically using the penalty method) to reproduce a 1D shear wave propagation mechanism effect. The 7.5 m depth (NS direction) downhole acceleration record (Figure 7) from the Wildlife Refuge site during the 1987 Superstition Hills earthquake was employed as base excitation along the x-axis.

Two cases were explored: one without any treatment (Medium-Sand, permeability $k = 6.6 \times 10^{-5}$ m/s) and the other one with remediation by compaction as well as high permeability effects (Dense Gravel, D_r = 65%, $k = 1.0 \times 10^{-2}$ m/s). The employed soil constitutive modeling parameters are summarized in Table 1.

Figure 8 shows the foundation vertical displacement time histories before and after remediation. The foundation final settlement was reduced to 0.21 m after remediation (compared to 0.28 m before remediation). Other more effective remediation strategies are reported in Lu (2005).

The simulations were performed on DataStar (IBM SP POWER4 machine) at SDSC using 64 processors. The total execution time for each simulation is about 18 hours.

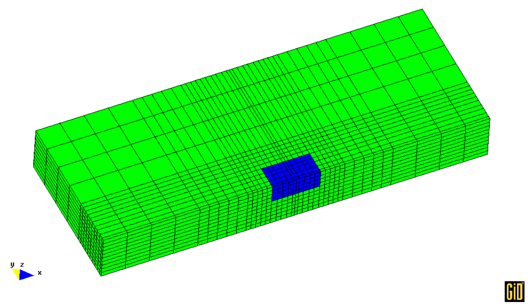

Figure 6. FE mesh of a shallow foundation model (dark zone shows the remediated area).

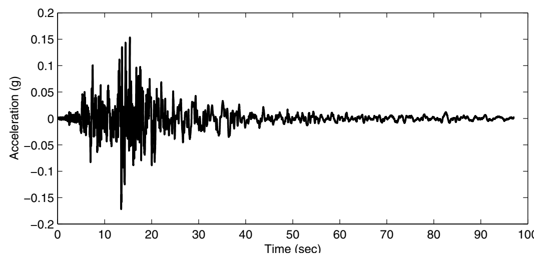

Figure 7. Base input motion.

Table 1. Model parameters for medium sand and dense gravel (Lu et al. 2005).

Parameters	Medium	Dense
Low-strain shear modulus G_r (at 80 kPa mean effective confinement)	78.5 MPa	135. MPa
Friction angle ϕ	31.4°	40.°
Figure 2, *phase 0-1)*		
Contraction parameter c_1	0.065	0.02
Contraction parameter c_2	400.	400.
*(*Figure 2*, phase 2-3)*		
Phase Transformation angle ϕ_{PT}	26.5°	26.°
Dilation parameter d_1	140.	200.
Dilation parameter d_2	1.	1.

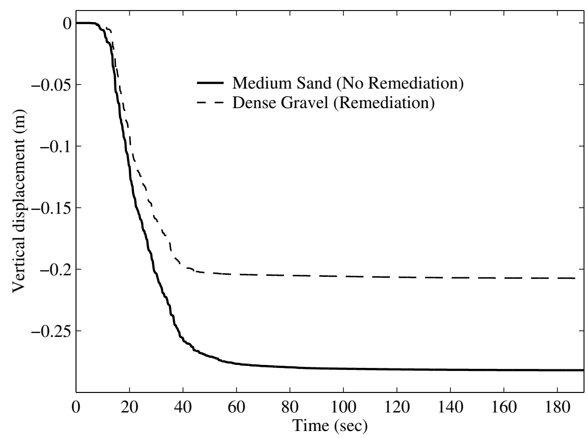

Figure 8. Foundation vertical displacement time histories before and after remediation.

Summary and Conclusions

This paper presents the analysis and solution strategies employed in ParCYCLIC, a parallel nonlinear finite element program for the simulation of earthquake site response and liquefaction. In ParCYCLIC, the calibrated serial code for modeling of earthquake geotechnical phenomena is combined with advanced computational methodologies to facilitate the simulation of large-scale systems and broaden the scope of practical applications. The parallel computational strategies employed in ParCYCLIC are general and can be adapted to other similar applications without difficulties. ParCYCLIC has been successfully ported on IBM SP machines, SUN super computers, and Linux workstation clusters.

A shallow foundation model was simulated using ParCYCLIC and preliminary results are presented. It is shown that ParCYCLIC can be used to simulate large-scale problems, which would otherwise be infeasible using single-processor computers due to limitations.

Acknowledgements

The reported research was supported by the Pacific Earthquake Engineering Research (PEER) Center, under the National Science Foundation Award Number EEC-9701568, and by the National Science Foundation (Grants No. CMS0084616 and CMS0200510). Additional funding was also provided by NSF through computing resources provided by San Diego Supercomputer Center (SDSC). This support is most appreciated.

References

Bielak, J., Hisada, Y., Bao, H., Xu, J., and Ghattas, O. (2000). "One- vs two- or three- dimensional effects in sedimentary valleys." *Proceedings of 12th World Conference on Earthquake Engineering*, New Zealand, February.

Chan, A. H. C. (1988). "A unified finite element solution to static and dynamic problems in geomechanics," PhD Thesis, University College of Swansea, U. K.

De Santiago, E., and Law, K. H. (1996). "A distributed finite element method for solving the incompressible navier-stokes equations." *International Journal for Numerical Methods in Engineering*, 39, 4243-4258.

Elgamal, A., Yang, Z., and Parra, E. (2002). "Computational modeling of cyclic mobility and post-liquefaction site response." *Soil Dynamics and Earthquake Engineering*, 22(4), 259-271.

Elgamal, A., Yang, Z., Parra, E., and Ragheb, A. (2003). "Modeling of cyclic mobility in saturated cohesionless soils." *International Journal of Plasticity*, 19(6), 883-905.

Ishihara, K., Tatsuoka, F., and Yasuda, S. (1975). "Undrained deformation and liquefaction of sand under cyclic stresses." *Soils and Foundations*, 15(1), 29-44.

Karypis, G., and Kumar, V. (1997). *METIS, a software package for partitioning unstructured graphs, partitioning meshes and compute fill-reducing ordering of sparse matrices*, Technical Report, Department of Computer Science, University of Minnesota.

Lade, P. V., and Duncan, J. M. (1975). "Elastoplastic stress-strain theory for cohesionless soil." *Journal of Geotechnical Engineering Division*, 101(GT10), 1037-1053.

Law, K. H., and Mackay, D. R. (1993). "A parallel row-oriented sparse solution method for finite element structural analysis." *International Journal for Numerical Methods in Engineering*, 36, 2895-2919.

Lu, J., Peng, J., Elgamal, A., Yang, Z., and Law, K. H. (2004). "Parallel finite element modeling of earthquake liquefaction response." *International Journal of Earthquake Engineering and Engineering Vibration*, 3(1), 23-37.

Lu, J. (2005). "Parallel finite element modeling of earthquake site response and liquefaction," PhD Thesis (in progress), Department of Structural Engineering, University of California, San Diego, La Jolla, CA.

Lu, J., Elgamal, A., and Yang, Z. (2005). "Pilot 3D numerical simulation of liquefaction and countermeasures." *Proceedings of ASCE Geo-Frontiers 2005*, Austin, TX, January 24-26.

Parra, E. (1996). "Numerical modeling of liquefaction and lateral ground deformation including cyclic mobility and dilation response in soil systems," PhD Thesis, Department of Civil Engineering, Rensselaer Polytechnic Institute, Troy, NY.

Peng, J., Lu, J., Law, K. H., and Elgamal, A. (2004). "ParCYCLIC: Finite element modeling of earthquake liquefaction response on parallel computers." *International Journal for Numerical and Analytical Methods in Geomechanics*, 28(12), 1207-1232.

Yang, Z. (2000). "Numerical modeling of earthquake site response including dilation and liquefaction," PhD Thesis, Department of Civil Engineering and Engineering Mechanics, Columbia University, New York, NY.

Yang, Z. (2002). "Development of geotechnical capabilities into OpenSees platform and their applications in soil-foundation-structure interaction analyses," PhD Thesis, Department of Civil Engineering, University of California, Davis, CA.

Yang, Z., and Elgamal, A. (2002). "Influence of permeability on liquefaction-induced shear deformation." *Journal of Engineering Mechanics*, 128(7), 720-729.

Yang, Z., Elgamal, A., and Parra, E. (2003). "A computational model for cyclic mobility and associated shear deformation." *Journal of Geotechnical and Geoenvironmental Engineering*, 129(12), 1119-1127.

Yang, Z., and Elgamal, A. (2004). "A multi-surface plasticity sand model including the Lode angle effect." *Proceedings of the 17th ASCE Engineering Mechanics Conference*, U. of Delaware, Newark, DE, June 13-16.