TECHNIQUES FOR SIMULATION OF LARGE-SCALE NONLINEAR SOIL-STRUCTURE SYSTEMS

Ahmed Elgamal[1], Jinchi Lu[1], Liangcai He[2], Kincho H. Law[3], and Zhaohui Yang[4]

Abstract: Parallel computing is gradually becoming a main-stream tool in geotechnical modeling. As an illustration, this paper presents numerical simulations of a large-scale pile-supported wharf system. These simulations are conducted on a supercomputer using a parallel nonlinear finite element program ParCYCLIC (recently developed based on the serial program CYCLIC). Ongoing efforts to calibrate CYCLIC are also presented. In this regard, data from large-scale shaking table experiments have been instrumental. On this basis, a user interface is under development, to allow numerical studies by interested researchers worldwide.

INTRODUCTION

Large-scale finite element (FE) simulations of soil-structure systems often require lengthy execution times. Utilization of parallel computers, which combine the resources of multiple processing and memory units, can potentially reduce the execution time significantly and allow simulations of large and complex models that may not fit into a single processor machine (Lu 2006).

Parallel computing is gradually becoming a main-stream tool in geotechnical simulations (e.g., Bielak et al. 2000; Yang 2002; Lu et al. 2004; Peng et al. 2004; Lu 2006). The need for high fidelity and for modeling of large 3-dimensional (3D) spatial configurations is motivating this direction of research.

A new parallel nonlinear finite element (FE) program ParCYCLIC (with implicit time integration employed) for seismic geotechnical applications has been recently developed. ParCYCLIC, is implemented based on the serial program CYCLIC, which is a nonlinear finite element program developed to analyze liquefaction-induced seismic response (Parra 1996; Yang and Elgamal 2002). Using ParCYCLIC, simulations of a large-scale 3D pile-supported wharf system were conducted and preliminary results are presented herein.

In the following sections, a brief description of CYCLIC and ParCYCLIC is presented. Recent calibration efforts based on large-scale shaking table experiments are also highlighted. Results from a 3D simulation of a wharf system are discussed. Finally, a user-interface for conducting routine analyses is shown to be a useful tool for practical applications.

CALIBRATION AND VALIDATION OF NUMERICAL FRAMEWORK

CYCLIC employs a two-phase (fluid and solid) fully-coupled FE formulation (Parra 1996; Yang and Elgamal 2002), based on the Biot theory (Biot 1962). In CYCLIC, the soil stress-strain behavior is governed by a new constitutive model (Yang 2000; Elgamal et al. 2002) within the general framework of multi-surface plasticity. Details of the FE framework

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[1] Department of Structural Engineering, University of California, San Diego, La Jolla, CA 92093
[2] Geomatrix Consultants, Inc., Newport Beach, CA 92663
[3] Department of Civil & Environment Engineering, Stanford University, Stanford, CA 94305
[4] URS Corporation, Oakland Office, Oakland, CA 94612

and the soil constitutive model employed in CYCLIC are included in Appendix A.

Calibration and validation has always been an integral part of CYCLIC development. Experimental programs conducted on the Rensselaer Centrifuge have been a major source of calibration over the years (Dobry et al. 1995; Elgamal et al. 1996; Dobry and Abdoun 1998). For instance, a large 2-dimensinal (2D) embankment liquefaction-countermeasure centrifuge investigation has been a key component (Adalier 1996; Parra 1996; Adalier et al. 1998).

Sample laboratory data were also used (Arulmoli et al. 1992; Parra et al. 1996; Yang 2000). In addition, valuable downhole-array earthquake data were used for calibration. This downhole-array calibration efforts started in the early 1990s and have been based on: i) Low (linear) and moderate (nonlinear) recorded earthquake data sets from Lotung (major set of 18 different earthquakes) and Hualien, Taiwan, as well as Treasure Island, CA (Elgamal et al. 1996; Gunturi et al. 1998), and ii) Liquefaction response data from Imperial County, California and Port Island, Kobe, Japan (Elgamal et al. 1996).

Current ongoing research includes calibration and validation based on a series of shake-table experiments recently conducted at the University of California, San Diego (UCSD) and the National Research Institute for Earth Science and Disaster Prevention (NIED) in Japan. To study the response of single piles and pile groups under liquefaction-induced lateral spreading conditions, these experiments employed a slightly inclined laminar box patterned after the work of Abdoun et al. (2003) and Dobry et al. (2003) to simulate lateral spreading in mild infinite slopes. Single piles of 0.10 to 0.32 m in diameter as well as 2x2 pile groups were tested in soil layers of up to 5.5 m depth with and without an upper non-liquefiable crust (He 2005).

Figures 1 & 2 depict the large and medium size laminar soil boxes (Kagawa et al. 2004; He 2005), respectively, employed in these recent experiments. Figure 3 shows the test setup and instrumentation of a single pile model (Model 7 in He 2005). Results of the shake-table experiments are employed for calibration of model parameters, through FE simulations (He 2005).

Figure 1: The NIED large size laminar box (Kagawa et al. 2004).

Figure 2: The UCSD medium size laminar box (Jakrapiyanun 2002).

SIMULATION OF PILE-SUPPORTED WHARF SYSTEM

The 3D model of a pile-supported wharf system was studied. The model configuration is based on typical geometries of pile-supported wharf structures (Berth 100 Container Wharf at the Port of Los Angeles). Figure 4 shows the idealized wharf model employed in this study. In Figure 4, a slice in this wharf system (central section) is shown, that exploits symmetry of the supporting pile-system configuration (Lu 2006).

There are 16 piles in 6 rows in this idealized model. Each pile is 0.6 m (24 inch) in diameter, and 43 m in length (reinforced concrete). The cracked flexural rigidity (**EI**) of the pile is 159 MN-m^2, with a moment of inertia (**I**) of 7.09 x 10^{-3} m^4. Relative to the piles, the wharf deck was assumed to be stiff in this model (with a thickness of 0.8 m).

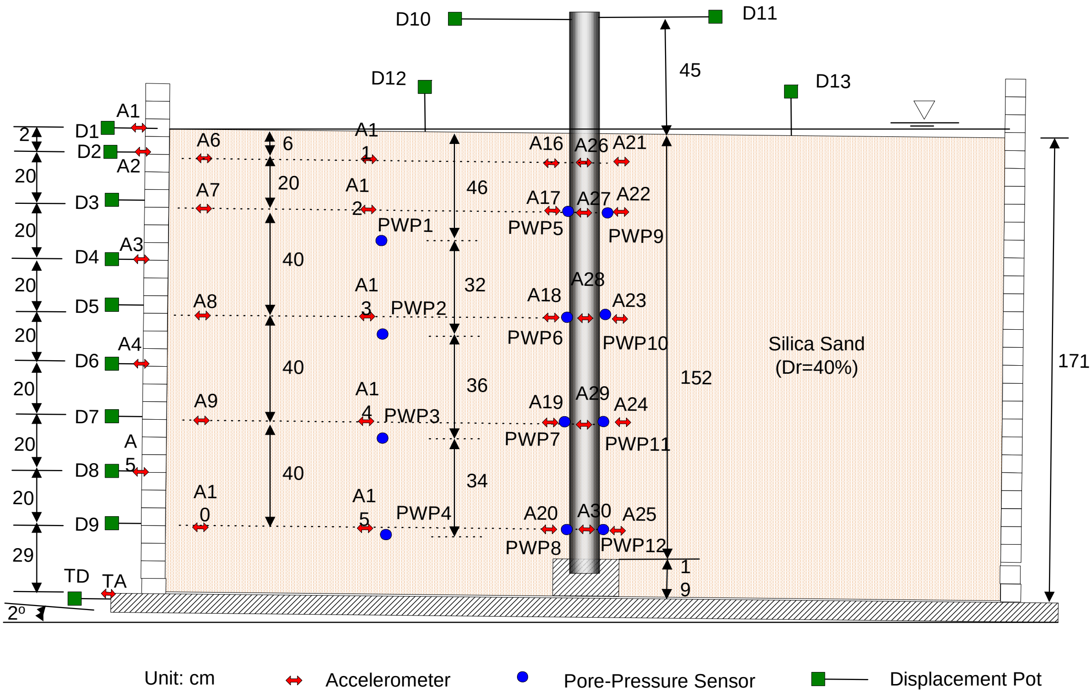

Figure 3: Test setup of Model 7 (He 2005).

Two soil layers were represented in this idealized model. The lower layer (25 m in height) is modeled as stiff clay (255 kPa of Cohesion) with the upper layer being a weaker medium-strength clay (44 kPa of Cohesion). Water table level was located at 16.6 m above the mud-line. The inclination angle of the slope was about 39 degrees.

The base of the FE model was assumed to be rigid (the actual bedrock level is much deeper at this site). A scaled Rinaldi Receiving Station record from the 1994 Northridge Earthquake was employed as the base input motion. On the waterside and landside of the FE model, motion was specified as the computed accelerations from a 1D shear beam simulation (Yang et al. 2004) of the left and right soil columns. Symmetry along the side boundaries was represented by roller supports.

Modeling of the above pile-supported wharf system was conducted using ParCYCLIC (see Appdendix B for details of the parallel implementation in ParCYCLIC) on the machine Datastar at the San Diego Supercomputer Center (SDSC). Datastar is SDSC's largest IBM terascale machine, built in a configuration particularly suited to data intensive computations. DataStar is composed of 272 (8-way) P655+ compute nodes, each with 8 POWER4 RISC-based processors and 16 GBytes of memory (SDSC 2006).

Simulation Results

Figure 5 shows the final deformed mesh of the pile-supported wharf system. The close-up view of the wharf section for the final deformed mesh is shown in Figure 6. As can be seen, the majority of the deformation occurs within the upper layer while the lower soil layer shows insignificant lateral displacement.

The computed wharf deck longitudinal displacement time history is shown in Figure 7, with a longitudinal displacement of about 0.3 m. In Figure 7, the large “jump”in displacement corresponds to the large acceleration phase (the near-fault “fling motion”) in the base input record (bottom graph in Figure 7).

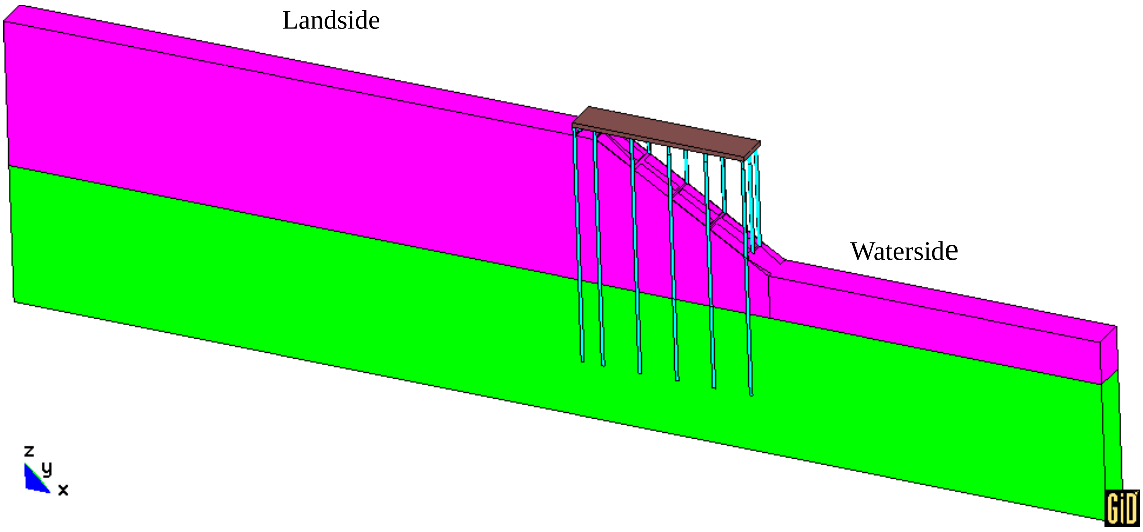

Figure 4: Pile-supported wharf model (Lu 2006).

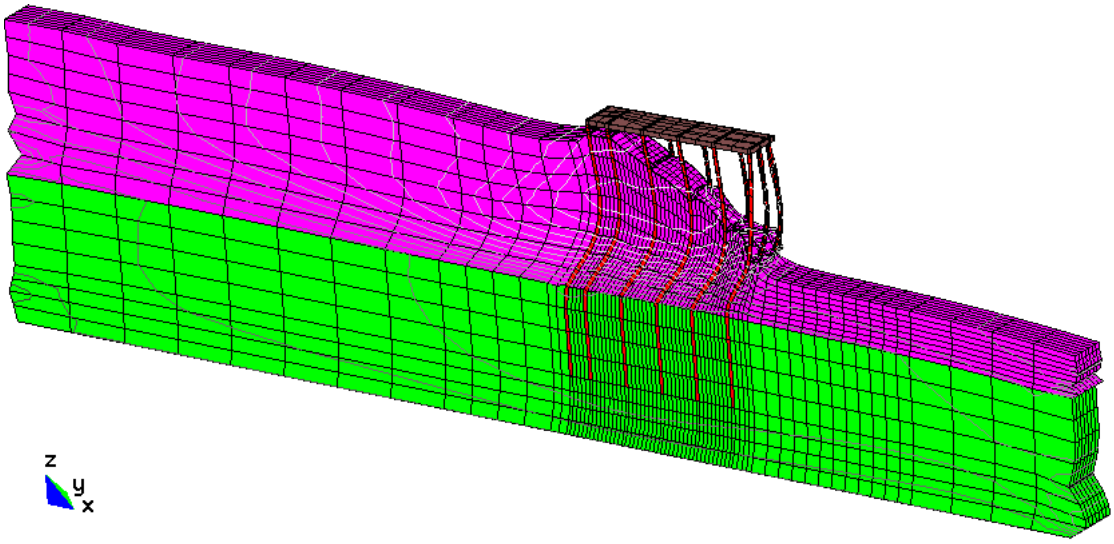

Figure 5: Final deformed mesh (factor of 30) (Lu 2006).

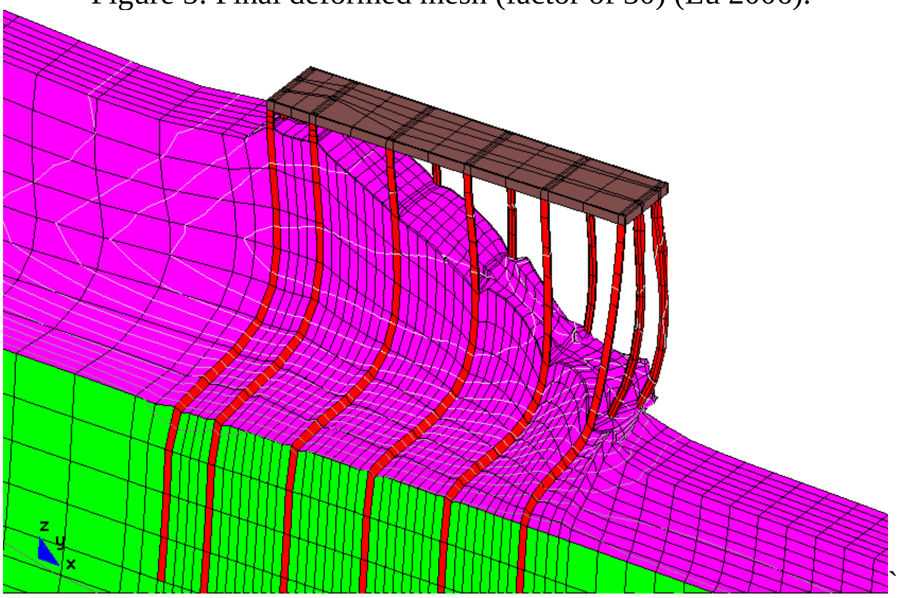

Figure 6: Close-up of final deformed mesh (factor of 30) (Lu 2006).

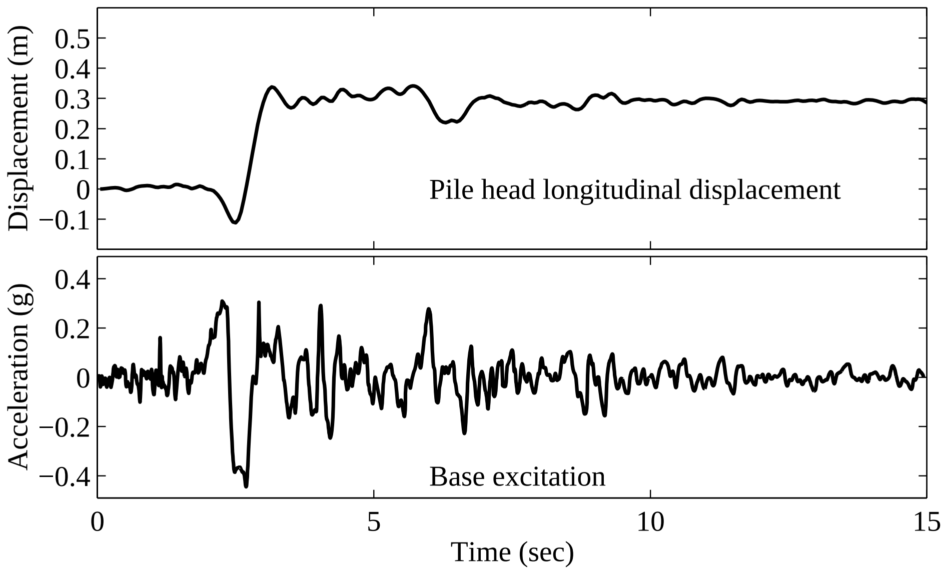

Figure 7: Deck longitudinal displacement time history and base input record (Lu 2006).

USER-INTERFACE

A user-interface "OpenSeesPL" is under development (Figure 8), to allow for the execution of seismic single-pile ground simulations as well as for push-over studies (Lu et al. 2006). Various ground modification scenarios may be also studied by appropriate specification of the material within the pile zone. The FE analysis engine for this interface is the OpenSees Framework (Mazzoni et al. 2006).

OpenSeesPL includes a pre-processor for: 1) definition of the pile geometry (circular or square pile) and material properties (linear or nonlinear), 2) definition of the 3D spatial soil domain, 3) definition of the boundary conditions and input excitation or push-over analysis parameters, and 4) selection of soil materials from an available menu of cohesionless and cohesive soil materials. The menu of soil materials includes a complementary set of soil modeling parameters representing loose, medium and dense cohesionless soils (with silt, sand or gravel permeability), and soft, medium and stiff clay (J_2 plasticity cyclic model).

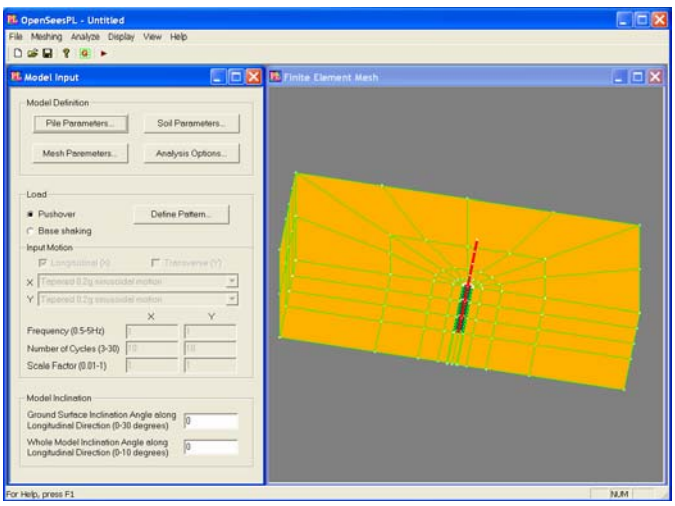

Figure 8: OpenSeesPL user interface with mesh showing a circular pile in level ground (view of ½ mesh shown due to symmetry for uni-directional lateral loading).

OpenSeesPL allows convenient pre-processing and graphical visualization of the analysis results including the deformed mesh (Figure 9), ground response time histories and pile responses. OpenSeesPL makes it possible for geotechnical and structural engineers/researchers to rapidly build a model, run the FE analysis, and evaluate performance of the pile-ground system (Lu et al. 2006).

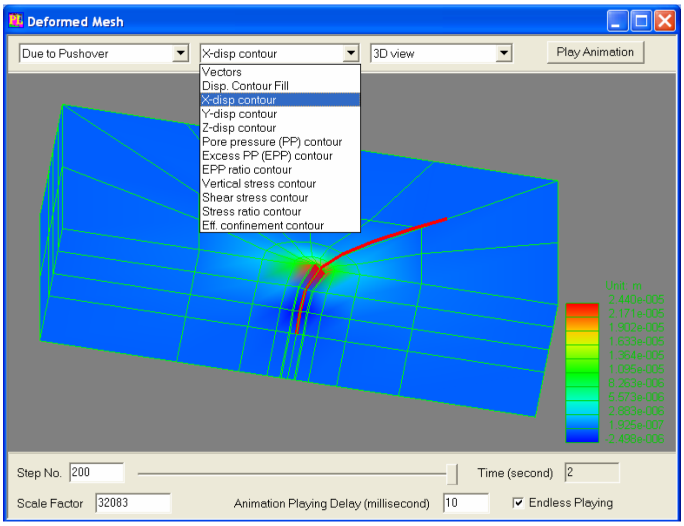

Figure 9: Graph types available in the deformed mesh window (Lu 2006).

SUMMARY AND CONCLUSIONS

This paper presents techniques for seismic simulation of large-scale soil-structure response. The reported simulations of a wharf system were conducted on a supercomputer using ParCYCLIC, a parallel nonlinear finite element program. ParCYCLIC allows for the simulation of earthquake site response and liquefaction. In ParCYCLIC, the calibrated serial code CYCLIC is combined with advanced computational methodologies to facilitate the simulation of large-scale systems and broaden the scope of practical applications. It is shown that ParCYCLIC can be used to simulate scenarios which would otherwise be infeasible using single-processor computers due to memory limitations. A user-interface was shown to be a useful tool for conducting routine analyses in this 3D environment.

ACKNOWLEDGEMENTS

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APPENDIX A: COMPUTATIONAL FRAMEWORK AND CONSTITUTIVE MODEL

In CYCLIC and ParCYCLIC, the saturated soil system is modeled as a two-phase material. A simplified numerical framework of this theory (Chan 1988), known as *u-p* formulation (in which displacement of the soil skeleton *u*, and pore pressure *p*, are the primary unknowns), was implemented in a 3D FE program CYCLIC (Parra 1996; Yang 2000; Yang and Elgamal 2002; Lu 2006).

The *u-p* formulation is defined by (Chan 1988): 1) the equation of motion for the solid-fluid mixture, and 2) the equation of mass conservation for the fluid phase that incorporates equation of motion for the fluid phase and Darcy's law. These two governing equations are expressed in the following finite element matrix form (Chan 1988):

$$\mathbf{M\ddot{U}} + \int_{\Omega} \mathbf{B}^{T}\boldsymbol{\sigma}'\, d\boldsymbol{\Omega} + \mathbf{Qp} - \mathbf{f}^{s} = \mathbf{0} \qquad (1a)$$

$$\mathbf{Q}^{T}\mathbf{\dot{U}} + \mathbf{S\dot{p}} + \mathbf{Hp} - \mathbf{f}^{p} = \mathbf{0} \qquad (1b)$$

where $\mathbf{M}$ is the total mass matrix, $\mathbf{U}$ the displacement vector, $\mathbf{B}$ the strain-displacement matrix, $\boldsymbol{\sigma}'$ the effective stress tensor, $\mathbf{Q}$ the discrete gradient operator coupling the solid and fluid phases, $\mathbf{p}$ the pore pressure vector, $\mathbf{S}$ the compressibility matrix, and $\mathbf{H}$ the permeability matrix. The vectors $\mathbf{f}^{s}$ and $\mathbf{f}^{p}$ represent the effects of body forces and prescribed boundary conditions for the solid-fluid mixture and the fluid phase, respectively. Equations 1a and 1b are integrated in the time domain using a single-step predictor multi-corrector scheme of the Newmark type (Chan 1988; Parra 1996).

Soil Constitutive Model

The soil constitutive model (Parra 1996; Yang 2000; Yang and Elgamal 2002) was developed based on the original multi-surface-plasticity theory for frictional cohesionless soils (Prevost 1985). This model (Figures 10 and 11) was developed with emphasis on simulating the liquefaction-induced shear strain accumulation mechanism in clean medium-dense sands (Yang and Elgamal 2002; Elgamal et al. 2003). Special attention was given to the deviatoric-volumetric strain coupling (dilatancy) under cyclic loading, which causes increased shear stiffness and strength at large cyclic shear strain excursions (i.e., cyclic mobility). The main modeling parameters include standard dynamic soil properties such as low-strain shear modulus and friction angle, as well as parameters to control the dilatancy effects (phase transformation angle, contraction, and dilation), and the level of liquefaction-induced yield strain (γ_y).

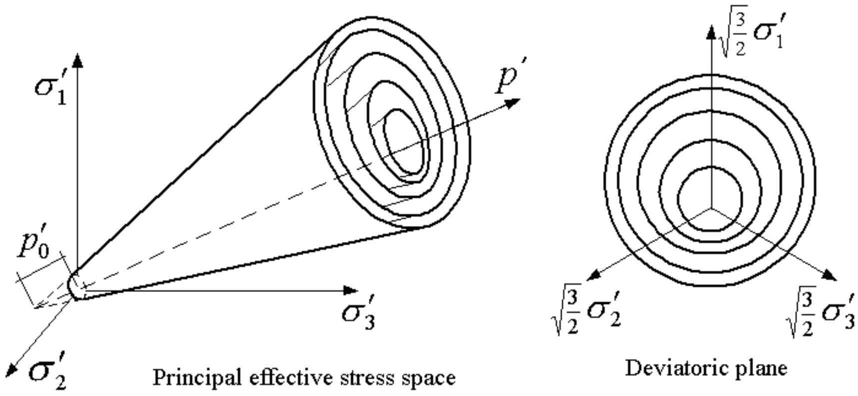

Figure 10: Conical yield surfaces for granular soils in principal stress space and deviatoric plane (after Prevost 1985; Yang et al. 2003).

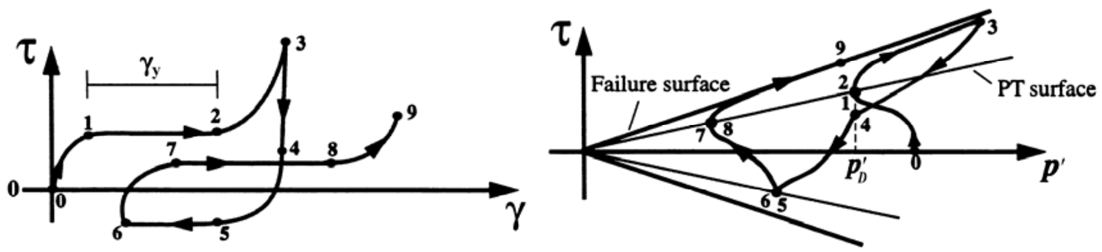

Figure 11: Shear stress-strain and effective stress path under undrained shear loading conditions (Yang et al. 2003).

APPENDIX B: PARALLEL IMPLEMENTATION

Key elements of the computational strategy employed in ParCYCLIC, designed for distributed-memory message-passing parallel computer systems, include (Lu et al. 2004; Peng et al. 2004; Lu 2006): (a) a parallel sparse direct solver (Law and Mackay 1993), in which LDL^T factorization is performed (where L is a unit lower triangular matrix and D is a diagonal matrix); (b) nodal ordering strategies to minimize storage space for the matrix coefficients; (c) an efficient scheme for the allocation of sparse matrix coefficients among the processors; and (d) an automatic domain decomposer, where METIS (Karypis and Kumar 1997) is used to partition the finite element mesh.

ParCYCLIC employes the single-program-multiple-data (SPMD) programming paradigm. Communication in ParCYCLIC is written in MPI (Snir and Gropp 1998), making ParCYCLIC portable, capable of running on a wide range of parallel computers and workstation clusters. ParCYCLIC has been successfully ported on IBM SP machines, SUN super computers, and Linux workstation clusters.

ParCYCLIC handles symmetric systems of linear equations (resulting from the employed implicit time integration scheme) using a parallel sparse solver (Law and Mackay 1993). This solver is based on a row-oriented storage scheme that takes full advantage of the sparsity of the stiffness matrix. The concept of the sparse solver is briefly described below (Lu et al. 2004; Peng et al. 2004; Lu 2006).

It is well known that the nonzero entries in the lower triangular matrix factor L can be determined by the original nonzero entries of the stiffness matrix K (Law and Fenves 1986) and a list vector, which is defined as:

$$PARENT(j) = \min\{i \mid L_{ij} \neq 0\} \qquad (2)$$

The array *PARENT* represents the row subscript of the first nonzero entry in each column of L. The definition of the list array *PARENT* results in a monotonically ordered elimination tree of which each node has its numbering higher than its descendants. By topologically post-ordering the elimination tree, the nodes in any subtree can be numbered consecutively. The resulting sparse matrix factor is partitioned into block submatrices where the columns/row of each block corresponds to the node set of a branch in the elimination tree. Figure 12 shows a simple square finite element grid and its post-ordered elimination tree representation.

The coefficients of a sparse matrix factor are distributively stored among the processors according to the column blocks. The strategy is to assign the rows corresponding to the nodes along each branch of the elimination tree (column block) to a processor or a group of processors. Beginning at the root of the elimination tree, the nodes belonging to this branch of the tree are assigned among the available processors in a rotating round robin fashion (Golub and Van Loan 1989). As we traverse down the elimination tree, at each fork of the elimination tree, the group of processors is divided to match the number and size of the subtrees below the current branch. A separate group of processors is assigned to each branch at the fork and the process is repeated for each subtree.

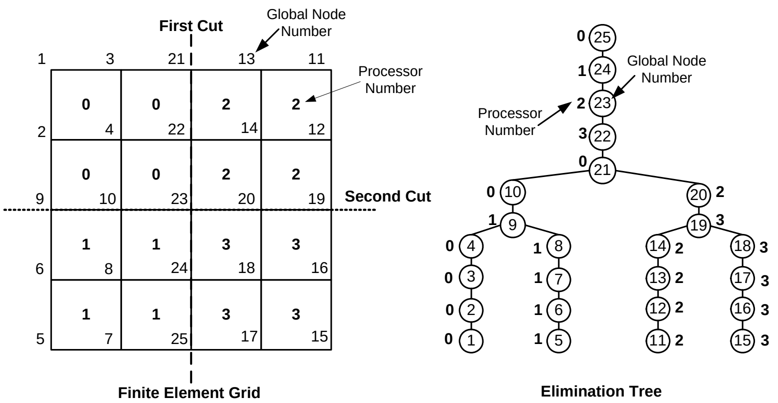

Figure 12. A finite element grid and its elimination tree representation (Peng et al. 2004).

The parallel numerical factorization procedure is divided into two phases (Law and Mackay 1993). In the first phase, each processor independently factorizes the portions of the matrix assigned to a single processor. In the second phase, other portions of the matrix shared by more than one processor are factored. Following the parallel factorization, the parallel forward and backward solution phases proceed to compute the solution to the global system of equations (Lu et al. 2004; Peng et al. 2004).