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PARCYCLIC: FINITE ELEMENT MODELING OF EARTHQUAKE LIQUEFACTION RESPONSE ON PARALLEL COMPUTERS

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SUMMARY

ParCYCLIC is a parallel nonlinear finite element program designed for the analysis of cyclic seismically induced liquefaction problems. An incremental plasticity formulation and a constitutive model developed for simulating liquefaction-induced deformations are main components of this analysis program. The parallel computational strategy of ParCYCLIC includes a parallel sparse direct solver. Application of ParCYCLIC to simulate 3-D geotechnical experimental models is demonstrated. Not only has good agreement been achieved between the computed and recorded results, excellent parallel performance and scalability have also been obtained on parallel computers with large number of processors.

INTRODUCTION

Soil liquefaction is a complex phenomenon that causes many damages during earthquakes. The use of finite element methods for the simulations of earthquake response and liquefaction effects requires significant amount of execution time due to the complexity involved in coupling solid and fluid, as well as the need for sophisticated soil plasticity models. Large-scale earthquake simulations often exceed the capacity of current single-processor computers. Utilization of parallel computers with multiple processing and memory units can potentially reduce the solution time and allow analysis of large and complex models. Sequential application software, such as traditional finite element programs, needs to be re-designed in order to take full advantage of parallel computers.

This research presents the implementation of a parallel version of a nonlinear finite element program for the simulation of earthquake ground response and liquefaction effects. The implementation is based on the serial program CYCLIC, which is a nonlinear finite element program for analyzing cyclic mobility and liquefaction problems [1, 2]. Extensive calibration of CYCLIC has been conducted with results from experiments and full-scale response of earthquake simulations involving ground liquefaction. CYCLIC program is re-designed and parallelized to form ParCYCLIC [3, 4]. A parallel sparse direct solver has

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been incorporated to improve the simulation performance [5]. The objectives of developing ParCYCLIC are to extend the computational capabilities of the finite element program to simulate large-scale systems, and to broaden the scope of its applications.

NUMERICAL MODELING OF PARCYCLIC

The numerical modeling of ParCYCLIC is based on the two phase (solid and fluid) fully-coupled finite element formulation of Chan [6] and Zienkiewicz et al. [7]. The soil constitutive model incorporated is developed specifically for liquefaction analysis [1, 8]. The current implementation is based on small-deformation theory and does not account for nonlinear effects due to finite deformation or rotation.

Finite Element Formulation

In ParCYCLIC, soil is modeled as a two-phase material, based on the concepts outlined by Biot [9] for fluid saturated porous media. ParCYCLIC adopts a simplified version of this theory [6, 7] that is suited to earthquake engineering applications, known as the *u-p* formulation (in which, the displacement of solid phase *u*, and pore fluid pressure *p*, are primary unknowns). The governing *u-p* formulation equation can be written as follows:

$$\nabla p + \mathbf{R} + \rho_f(\ddot{\mathbf{u}} - \mathbf{g}) = \mathbf{0} \tag{1}$$

where p is pore fluid pressure, $\mathbf{R}$ is viscous drag force vector (exerted on fluid by solid phase), ρ_f is fluid mass density, $\ddot{\mathbf{u}}$ is solid phase acceleration vector, and $\mathbf{g}$ is gravity acceleration vector. Using D'Arcy's law and invoking mass conservation of the mixture, Equation 1 leads to:

$$\dot{p}/Q + \nabla\cdot\dot{\mathbf{u}} - \nabla\cdot\left[\mathbf{k}\cdot(\nabla p + \rho_f\ddot{\mathbf{u}} - \rho_f\mathbf{g})/\rho_f g\right] = 0 \tag{2}$$

where Q is bulk modulus of the mixture, and $\mathbf{k}$ is D'Arcy's permeability coefficient tensor. The mixture equation of motion can be written as:

$$\nabla\cdot(\boldsymbol{\sigma} - p\boldsymbol{\delta}) - \rho(\ddot{\mathbf{u}} - \mathbf{g}) = \mathbf{0} \tag{3}$$

where σ is the effective stress tensor, δ is identity tensor, and ρ is mass density of the mixture. Equations 2 and 3 constitute the *u-p* formulation. Within the finite-element framework, the resulting matrix equations are integrated in time using a single-step predictor, multi-corrector scheme of the Newmark type [1].

Soil Constitutive Model

In ParCYCLIC, the soil stress-strain behavior is governed by a new constitutive model [1, 8, 10] within the general framework of multi-yield surface plasticity for frictional, cohesionless soils. In the new model, emphasis is placed on modeling the cyclic stress-strain characteristics associated with shear-volumetric coupling during the post-liquefaction phase. The constitutive equation is written in incremental form as follows [11]:

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p) \tag{4}$$

where $\dot{\boldsymbol{\sigma}}$ is the rate of effective Cauchy stress tensor, $\dot{\boldsymbol{\varepsilon}}$ is the rate of deformation tensor, $\dot{\boldsymbol{\varepsilon}}^p$ is the plastic rate of deformation tensor, and $\mathbf{E}$ is the isotropic elastic coefficient fourth order tensor. The yield function f is defined as [11, 12]:

$$f = \frac{3}{2}(\mathbf{s} - p_a\boldsymbol{\alpha}) : (\mathbf{s} - p_a\boldsymbol{\alpha}) - M^2 p_a{}^2 = 0 \tag{5}$$

where $\mathbf{s} = \boldsymbol{\sigma} - p\boldsymbol{\delta}$ is the deviatoric stress tensor, $p_a = p - a$ with p and a representing the effective mean normal stress and the residual shear strength at the apex respectively, $\boldsymbol{\alpha}$ is the kinematic deviatoric tensor defining the coordinates of the yield surface center in deviatoric stress subspace, and M is a material parameter related to the soil friction angle ϕ. A family of nested conical surfaces defines the hardening zone in stress space, as shown in Figure 1.

In order to characterize the effects on volume change exhibited by granular materials during shear loading, it is necessary to employ a non-associative flow rule. Typically, non-associativity is restricted to the volumetric component of the plastic flow so as to control the dilation/contraction effects (mean effective stress) associated with shear loading (in accordance with laboratory observations). As shown in Figure 2, the plastic flow is defined separately for stress states above, on, or below the phase transformation (PT) surface using a phenomenological approach.

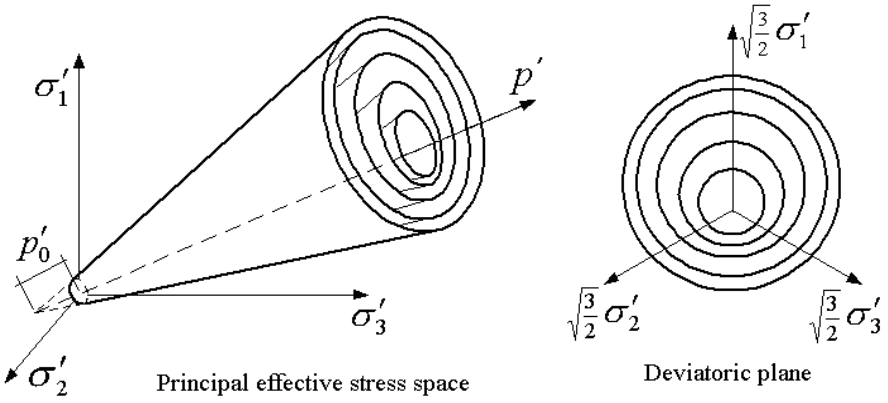

Figure 1. Conical Yield Surfaces for Granular Soils in Principal Stress Space and Deviatoric Plane

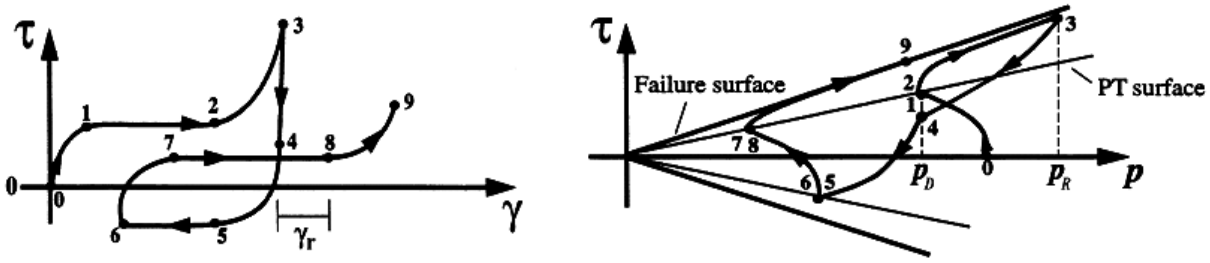

Figure 2. Constitutive Model and Effective Stress Path under Undrained Shear Loading

PARALLEL IMPLEMENTATION

One common approach in developing application software for distributed memory parallel computers is to use the single-program-multiple-data (SPMD) paradigm. In this parallel programming paradigm, all processors are assigned the same program code but run with different data sets comprising the problem. Each processor of the parallel machine solves a partitioned domain, and data communications among sub-domains are performed through message passing. The inter-processor communication of ParCYCLIC is implemented using MPI (Message Passing Interface) [13], which is a specification of a standard library for processors to exchange messages.

Computational Procedures

The computational procedure of ParCYCLIC is illustrated in Figure 3. The procedure can be divided into three phases: the initialization phase, the nonlinear solution phase, and the postprocessing phase. The initialization phase consists of reading input files, performing mesh partitioning and symbolic factorization. METIS [14], which is a set of libraries for graph partitioning developed at the University of Minnesota, is used to partition the finite element mesh at this phase. Specifically, the multilevel nested dissection algorithm in METIS is employed to partition the finite element mesh.

The symbolic factorization is performed after the initialization phase to determine the nonzero pattern of the matrix factor. The storage space for the matrix factors is also allocated during this phase. Since all the processors need to know the nonzero pattern of the global stiffness matrix and symbolic factorization generally only takes a small portion of the total runtime, the domain decomposition and symbolic factorization are performed within each processor based on the global input data.

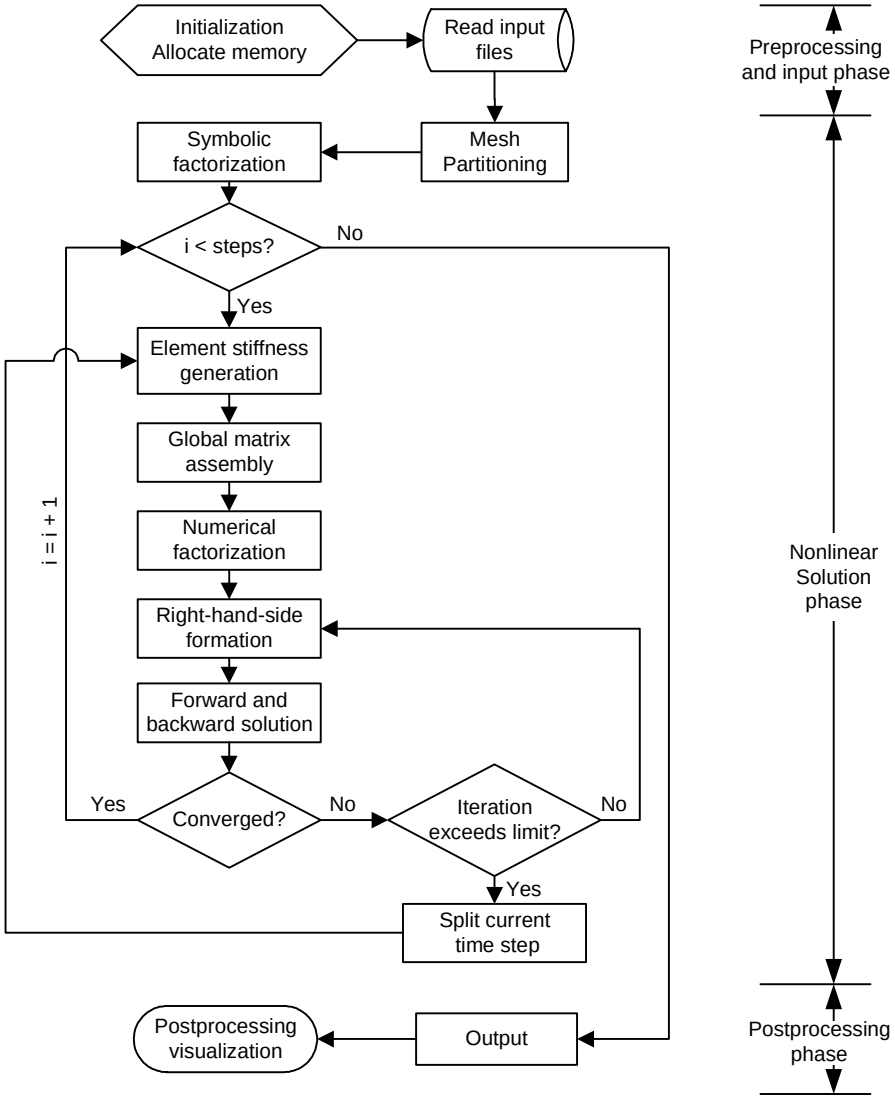

Figure 3. Flowchart of Computational Procedures in ParCYCLIC

In the nonlinear analysis solution phase, the program essentially goes through a while loop until the number of increments reaches the pre-set limit. In the nonlinear solution phase, the modified Newton-Raphson algorithm is employed, i.e., the stiffness matrix at each iteration step uses the same tangential stiffness from the initial step of the increment. The convergence test is performed at the end of each iteration step. If the solution is not converged after a certain number of iterations (e.g., 10 iterations) within a particular time step, the time step will be divided into two to expedite convergence. This process repeats until the solution converges.

The final phase, output and postprocessing, consists of collecting the calculated node response quantities (e.g. displacements, acceleration, pore pressure, and etc.) and element output (includes normal stress, normal strain, volume strain, shear strain, mean effective stress, and etc.) from the different processors. The response quantities and timing results are then written into files for future processing and visualization.

Parallel Sparse Direct Solver

Nonlinear finite element computations of earthquake simulations involve the iterative solution of sparse symmetric systems of linear equations. Solving the linear system is often the most computational intensive task of a finite element program, especially when an implicit time integration scheme is employed. The parallel sparse solver implemented in ParCYCLIC is based on a row-oriented storage scheme that takes full advantage of the sparsity of the stiffness matrix [4, 5]. A direct solver is preferred in ParCYCLIC over an iterative solver because even the best-known iterative solver (e.g. the Polynomial Preconditioned Conjugate Gradient method (PPCG)) may exhibit instabilities under certain conditions. For instance, in a nonlinear analysis, an iterative solver may diverge [15]. The direct solution method is a more stable approach to achieve solution convergence.

Given a linear system of equations $Kx = f$, the symmetric sparse matrix K is often factored into the matrix product LDL^T, where L is a lower triangular matrix and D is a diagonal matrix. The solution vector x is then computed by a forward solution, $Lz = f$ or $z = L^{-1}f$, followed by a backward substitution $DL^Tx = z$ or $x = L^{-T}D^{-1}z$. In the parallel sparse direct solver, each processor only holds a portion of the global matrix K. The sparse matrix factorization can be divided into two phases: symbolic factorization and numeric factorization. Symbolic factorization determines the structure of matrix factor L from that of K (i.e. locations of nonzero entries). Numeric factorization then makes use of the data structure determined to compute the numeric values of L and D.

The parallel numerical factorization procedure is divided into two phases. In the first phase, each processor independently factorizes certain portions of the matrix that assigned to a single processor. In the second phase, other portions of the matrix shared by more than one processor are factored. Following the parallel factorization, the parallel forward and backward solution phases proceed to compute the solution to the global system of equations. Since each processor only has part of the global solution vector that it is responsible for, messages are needed to gather the global solution from each processor.

NUMERICAL SIMULATION OF CENTRIFUGE EXPERIMENTS

ParCYCLIC is used to perform earthquake simulations for three-dimensional geotechnical models on the Blue Horizon machine at San Diego Supercomputer Center. Blue Horizon is an IBM Scalable POWERparallel (SP) machine which has 144 compute nodes, each with 8 POWER3 RISC-based processors and with 4 GBytes of memory. Each processor on the node has equal shared access to the memory. The following presents the simulations of two 3D centrifuge test models.

Stone Column Centrifuge Test Model

The first example is a stone-column centrifuge test model. As shown in Figure 4, only half of the mesh is used due to its geometrical symmetry. Many of the past earthquake-induced ground failures and large deformations observed in the built-environment have occurred mostly in sites containing non-plastic silty soils. One of the improved remediation techniques to mitigate liquefaction hazards in these silty soils is installing stone columns [16]. In the stone column test model, a number of gravel columns are embedded into a fully-saturated soil foundation filled with silt. The model is then subjected to earthquake excitation along the x-direction at the base.

Since the simulation of this model requires significant computer resources, only one time step is performed to show the parallel performance of ParCYCLIC. Table 1 summarizes the timing results of the solution phase, the LDL^T numerical factorization, the forward and backward solutions, and the total execution time (which includes the initialization phase) for one time step. Note that the stone column model, with a scale of 364,800 degrees of freedom (dofs), cannot fit into the memory of less than 4 processors. As shown in Table 1, excellent parallel speedup is achieved for this model.

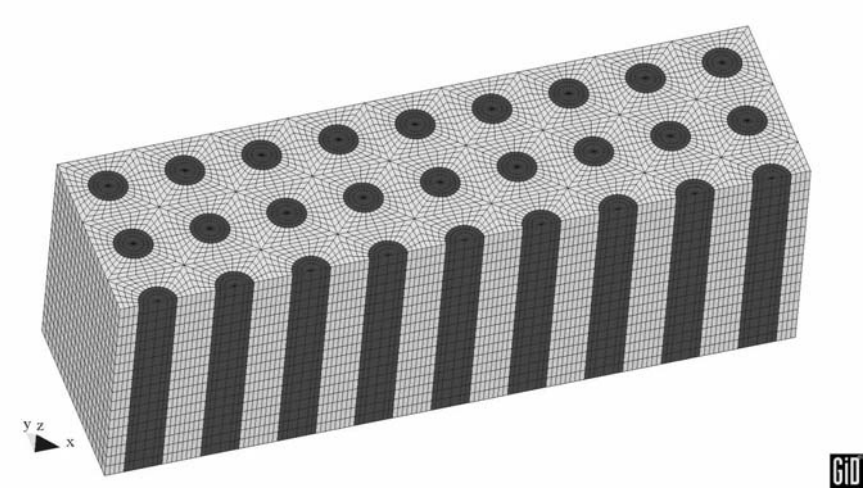

Figure 4. Finite Element Model of the Stone Column Centrifuge Test

Table 1. Solution Times for the Stone Column Centrifuge Test Model (time in seconds)

Number of processors	LDL^T factorization	Forward and backward solve	Solution phase	Total execution time
4	1246.08	2.76	1306.87	1769.00
8	665.66	1.56	702.09	1150.17
16	354.99	0.98	378.35	841.38
32	208.90	0.67	225.93	668.02
64	125.05	0.66	142.33	583.98

Centrifuge Test Model for Mildly Sloping Sand Sites

The second experiment is a centrifuge test model investigated by Dobry et al. [17] to simulate the dynamic response of mildly sloping sand sites. As shown in Figure 5, the model represents a mildly inclined infinite slope with an inclination angle of 2°, subjected to a predominantly 2 Hz harmonic base excitation. The test was performed in a laminated container that allows relative slip between laminates in

order to simulate approximately one-dimensional (1D) shear response. Nevada sand was used at *Dr* in the range of 40-45%. The soil models were spun to a 50g gravitational field [18]. At this gravitational field, the centrifuge models aim to simulate a prototype stratum of 22.86m long, 12.70m wide and 10m high. Water was used as the pore fluid, resulting in prototype soil permeability equal to 50 times that of the model soil.

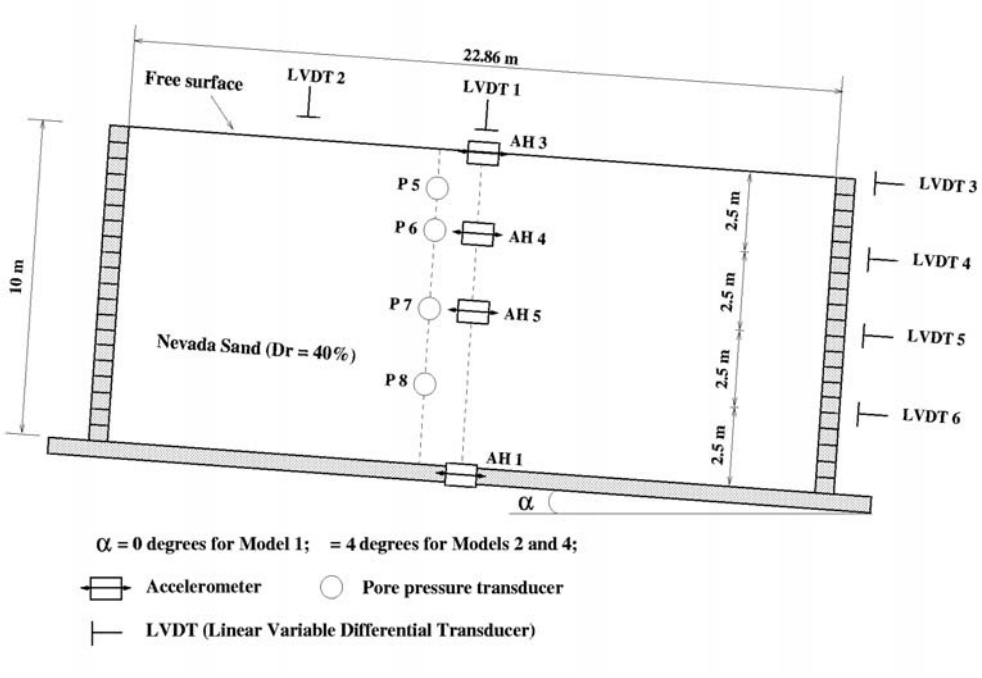

Figure 5. General Configurations of Test Model in Laminar Container

A grid mesh with 60 by 30 by16 elements (128,588 degrees of freedom in total) was used for the simulations. The boundary conditions were: (1) dynamic excitation was defined as the recorded base acceleration, (2) at any given depth, displacement degrees of freedom of the downslope and upslope boundaries were tied together (both horizontally and vertically using the penalty method) to reproduce a 1D shear beam effect [1], (3) the soil surface was traction free, with zero prescribed pore pressure, and (4) the base and lateral boundaries were impervious. A static application of gravity (model own weight) was performed before seismic excitation. The resulting fluid hydrostatic pressures and soil stress-states served as initial conditions for the subsequent dynamic analysis.

The simulations were conducted using 32 processors on the Blue Horizon machine at San Diego Supercomputer Center. The total execution time was about 7 hours. Note that these simulations cannot fit into a single processor unit due to the limitation of memory size. As shown in Figure 6 and Figure 7, good agreement has been achieved between the computed and the recorded lateral acceleration and pore pressure responses. Salient liquefaction response characteristics, including acceleration spikes and excess pore pressure generation and dissipation, were captured with reasonable accuracy. Both the recorded and computed excess pore pressure histories (Figure 7) displayed a number of instantaneous sharp pore pressure drops after initial liquefaction. These drops coincided with the observed and computed acceleration spikes that occurred exclusively in the negative direction (Figure 6).

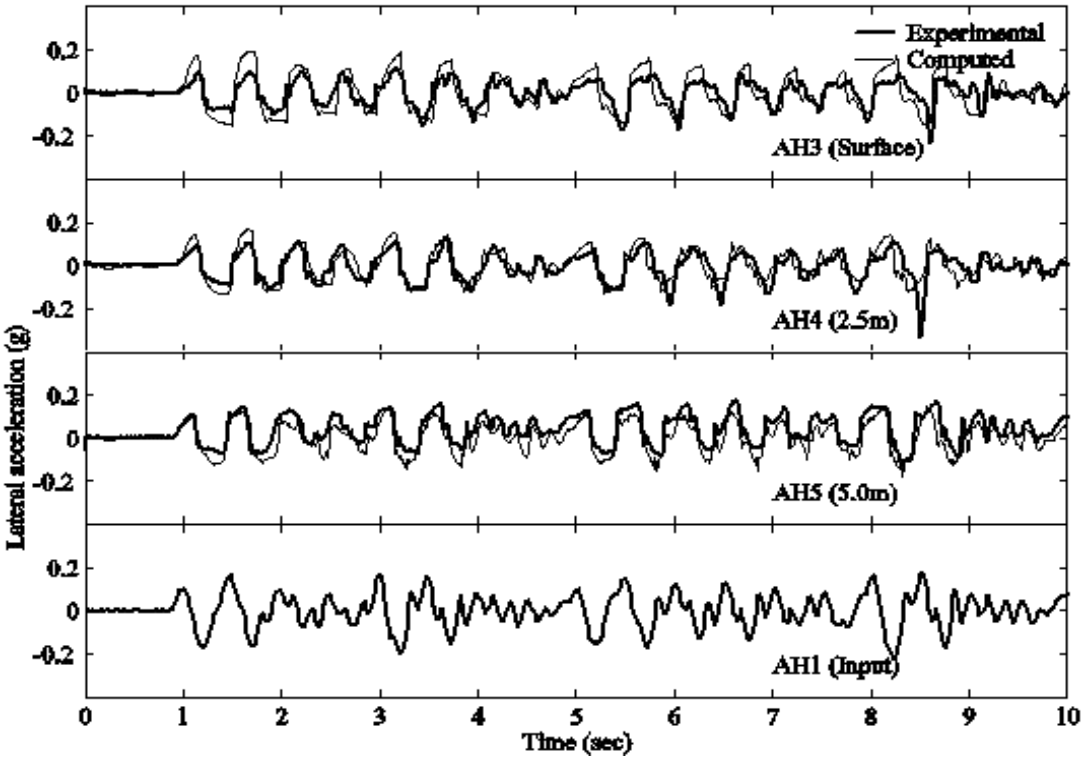

Figure 6: Recorded and Computed Lateral Acceleration Time Histories

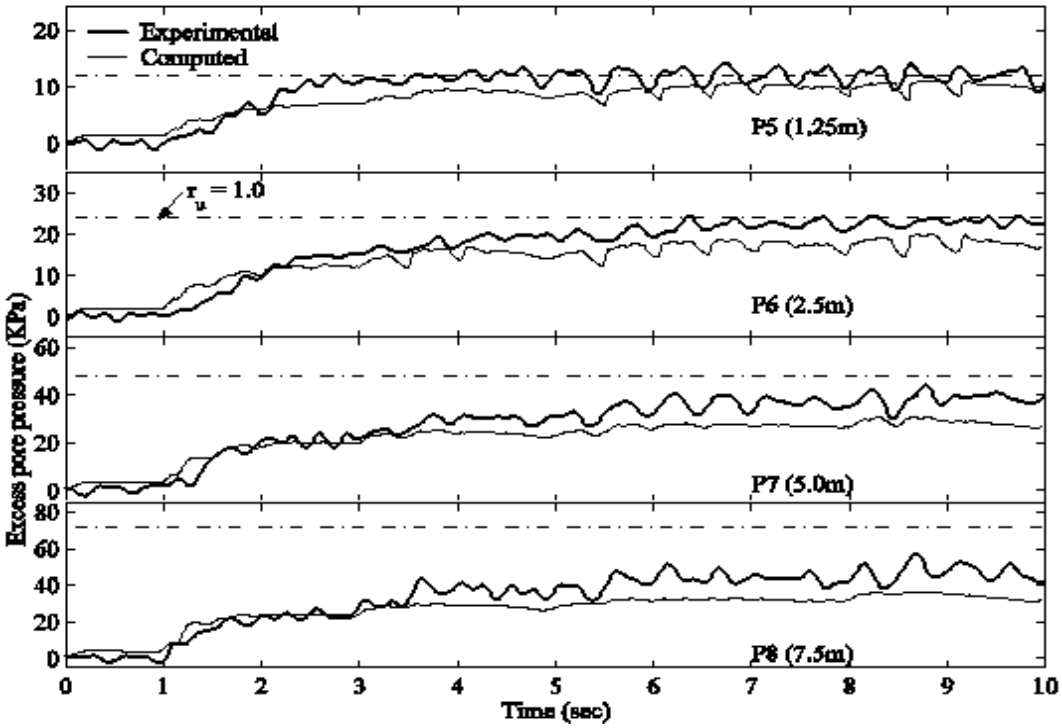

Figure 7: Recorded and Computed Excess Pore Pressure Time Histories

CONCLUSIONS

This paper presents the analysis and solution strategies employed in ParCYCLIC, a parallel nonlinear finite element program for the simulations of earthquake site response and liquefaction. In ParCYCLIC, finite elements are employed within an incremental plasticity, coupled solid-fluid formulation. A constitutive model developed for the simulation of liquefaction-induced deformations is a main component of this analysis framework. Extensive calibration of ParCYCLIC has been conducted with results from experiments and full-scale response of earthquake simulations involving ground liquefaction.

Large-scale experimental results for 3-D geotechnical simulations are presented to demonstrate the capability and the performance of ParCYCLIC. Good agreement has been achieved between the computed and the recorded acceleration and pore pressure responses. Excellent parallel speedups are reported from the simulation results. Furthermore, the results show that ParCYCLIC is scalable to a large number of processors, e.g., 64 or more. It is also demonstrated that ParCYCLIC can be used to simulate large-scale problems, which would otherwise be infeasible using single-processor computers due to the limited memory.

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